Pipes and cisterns: think in rates

A cistern is a tank. Filling the entire tank is the whole work.

Key ideas:

- Inlet: fills, positive rate.
- Outlet: drains, negative rate.
- Leak: unintended outlet, also negative.
- Assume each pipe has a constant rate, whether alone or together.

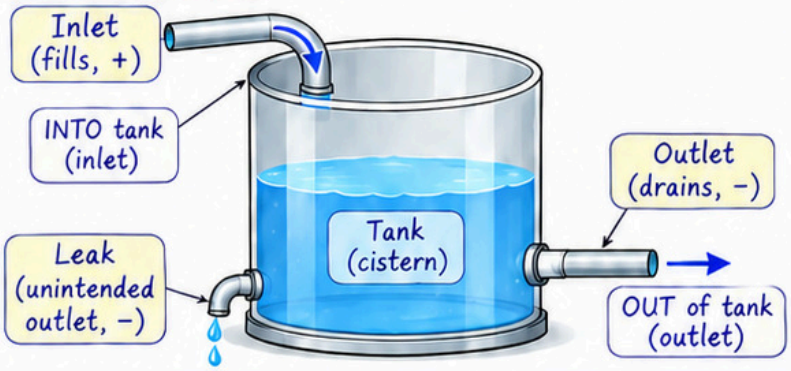
Rates of individual pipes

- An inlet fills in a hours: $+\frac{1}{a}$ tank per hour.
- An outlet empties in b hours: $-\frac{1}{b}$ tank per hour.

Net rate

NET RATE = sum of inlet rates − sum of outlet rates.

A tank with pipes



Rate balance (example form)

One inlet and one outlet:

$$\text{Net rate} = \frac{1}{a} - \frac{1}{b}.$$

Sign of the net rate and what happens

Positive net rate (> 0): an empty tank fills.



Zero net rate (= 0): water level does not change.



Negative net rate (< 0): an empty tank never fills; a full tank empties.



Time to fill, empty or change

$$\text{Time} = \frac{\text{quantity remaining}}{\text{net rate in the required direction.}}$$

- Use consistent units: hours with hourly rates, minutes with minute rates.

Efficiency

Efficiency = units per unit time.

Total work

Total work = capacity.

Identical filling pipes

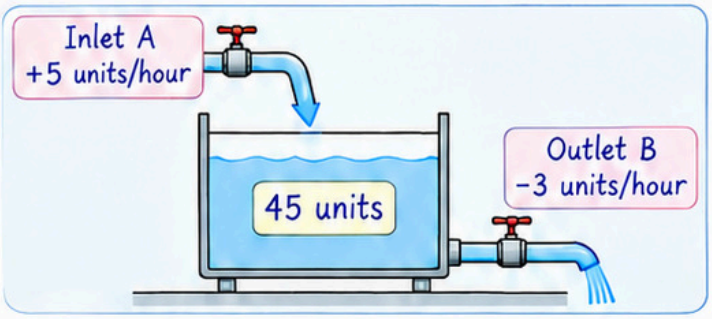
- Identical filling pipes: number of pipes × time = constant.
- More identical pipes need less time for the same tank.

Add or subtract RATES, never the filling times.

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One tank, two equivalent methods

Example: inlet A fills an empty tank in 9 hours, and outlet B empties the full tank in 15 hours. Both are opened together on the empty tank.



Step	Rate method (fraction method)	LCM (capacity) method
Total work	The whole tank = 1	Capacity = the LCM of 9 and 15 = 45 units
Inlet A	$+\frac{1}{9}$ of the tank an hour	$45 \div 9 = +5$ units an hour
Outlet B	$-\frac{1}{15}$ of the tank an hour (minus: it empties)	$-(45 \div 15) = -3$ units an hour
Net rate	$\frac{1}{9} - \frac{1}{15} = \frac{5}{45} - \frac{3}{45} = \frac{2}{45}$	$5 - 3 = 2$ units an hour
Time to fill	$1 \div \frac{2}{45} = \frac{45}{2} = 22\frac{1}{2}$ hours	$45 \div 2 = 22\frac{1}{2}$ hours

- The two columns are the same arithmetic: 45 is the common denominator the rate method needs, and the LCM method takes it at the start. The answer is 22 hours 30 minutes.
- The LCM method is the standard aptitude method. The steps are: find the LCM of the given times, take it as the total work in units, and take efficiency = total work ÷ time. For a tank, the only change is the sign of an outlet.
- Sense check. With an outlet open, the tank takes longer than the inlet alone (9 hours). The answer must also come out positive; if it does not, the outlet is the faster pipe.

Rate method (fraction method)

$$\text{Net rate} = \frac{1}{9} - \frac{1}{15} = \frac{5}{45} - \frac{3}{45} = \frac{2}{45} \text{ of the tank an hour}$$

$$\text{Time} = 1 \div \frac{2}{45} = \frac{45}{2} = 22\frac{1}{2} \text{ hours}$$

LCM (capacity) method

Capacity = LCM(9, 15) = 45 units

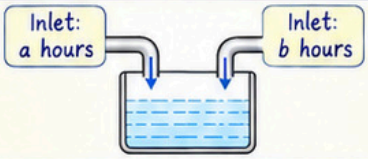
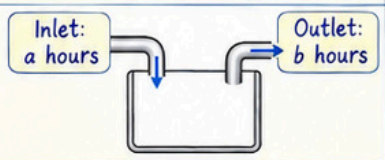
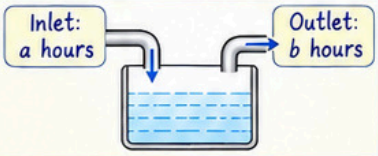
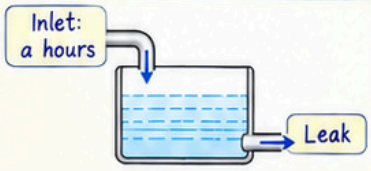
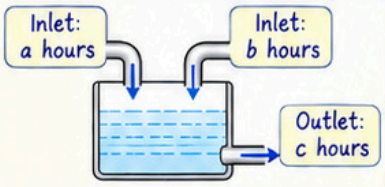
Rate of A = $45 \div 9 = +5$ units an hour

Rate of B = $-(45 \div 15) = -3$ units an hour

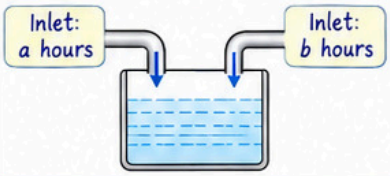
Net rate = $5 - 3 = 2$ units an hour

Time = $45 \div 2 = 22\frac{1}{2}$ hours

Formulas: signs decide the result

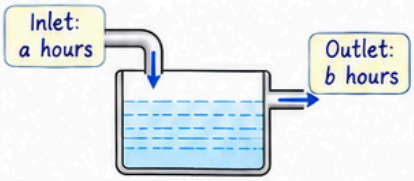
Situation	Result	Why, in one line
Two inlets, a hours and b hours 	Together: $\frac{ab}{a+b}$ hours	Rates add: $\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$, and the time is 1 divided by the rate
Inlet a hours, outlet b hours, b greater than a, tank empty 	Fills in $\frac{ab}{b-a}$ hours	Rates subtract: $\frac{1}{a}-\frac{1}{b}=\frac{b-a}{ab}$
Inlet a hours, outlet b hours, b less than a 	An empty tank never fills; a full tank empties in $\frac{ab}{a-b}$ hours	The outlet is faster, so the net rate $\frac{1}{b}-\frac{1}{a}$ is emptying
A pipe fills in a hours alone, but takes t hours longer because of a leak 	The leak alone empties the full tank in $\frac{a(a+t)}{t}$ hours	Leak's rate $=\frac{1}{a}-\frac{1}{a+t}=\frac{t}{a(a+t)}$
Two inlets (a and b hours) and an outlet (c hours) 	$\frac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}$ hours, when the net filling rate is positive	Add the inlets, subtract the outlet

Compare: two inlets



Both add.
Rate: $\frac{1}{a}+\frac{1}{b}$.
Time: $\frac{ab}{a+b}$ hours.

Compare: inlet/outlet



Signs differ.
Rate: $\frac{1}{a}-\frac{1}{b}$.
Time: $\frac{ab}{b-a}$ hours (if $b > a$).

Formula cards (with conditions)

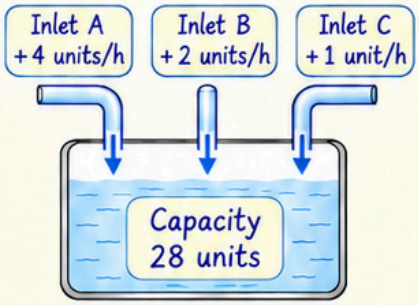
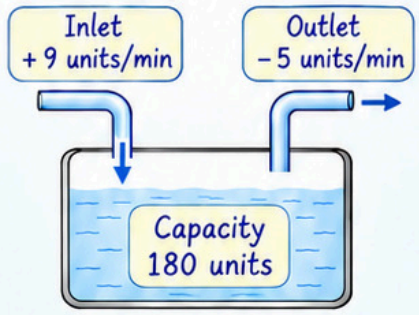
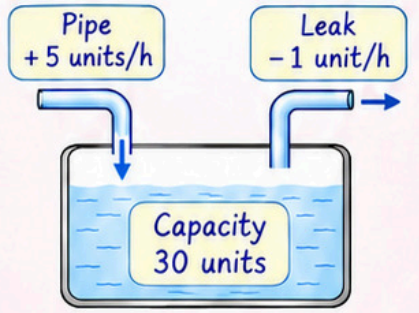
$\frac{ab}{a+b}$ hours	Two inlets of a hours and b hours (both add).
$\frac{ab}{b-a}$ hours	Inlet a hours, outlet b hours, b greater than a (fills).
$\frac{ab}{a-b}$ hours	Inlet a hours, outlet b hours, b less than a (a full tank empties).

Examples

- Two inlets of 14 hours and 35 hours fill the tank together in $14 \times 35 \div 49 = 10$ hours.
- The inlet and outlet of 2.2: $9 \times 15 \div 6 = 22\frac{1}{2}$ hours, as before.
- A pipe fills a tank in 4 hours, but a leak makes it take 1 hour longer: the leak alone empties the full tank in $4 \times 5 \div 1 = 20$ hours. Check: $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$.
- A full tank has an outlet that empties it in 6 hours and an inlet that fills it in 10 hours. With both open it empties in $6 \times 10 \div 4 = 15$ hours.
- If b is the time taken with the leak, then $b = a + t$, so the same result can also be written $\frac{ab}{b-a}$.
- For three or more pipes, the LCM method is quicker and safer than any formula.

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Three basic worked cases

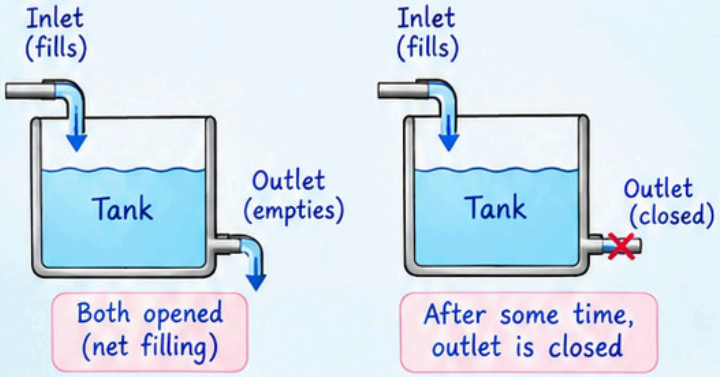
Type	Example	Working (LCM method)	Answer
Inlets together	Three inlets fill a tank in 7, 14 and 28 hours. 	Capacity 28 units. Rates: 7 hours → 28 ÷ 7 = 4 units/h 14 hours → 28 ÷ 14 = 2 units/h 28 hours → 28 ÷ 28 = 1 unit/h Total rate: 4 + 2 + 1 = 7 units an hour.	28 ÷ 7 = 4 hours.
Inlet and outlet	An inlet fills in 20 minutes and an outlet empties in 36 minutes; both are opened on the empty tank. 	Capacity 180 units. Rates: 20 minutes → 180 ÷ 20 = 9 units/min 36 minutes → 180 ÷ 36 = 5 units/min (the outlet, so −5 units/min) Net rate: 9 − 5 = 4 units a minute.	180 ÷ 4 = 45 minutes.
Leak	A pipe fills a tank in 6 hours; with a leak it takes $7\frac{1}{2}$ hours. How long does the leak alone take to empty the full tank? 	Capacity 30 units. The pipe alone: 30 ÷ 6 = 5 units/h. With the leak: $30 \div 7\frac{1}{2} = 30 \div \frac{15}{2} = 4$ units/h. So the leak takes away: 5 − 4 = 1 unit/h.	30 hours.

A pipe closes; another takes turns

Key idea

- Work by a pipe = rate × time open.
- First find the filled quantity; divide only what remains by the new rate.
- Distinguish extra time from the time since the start.
- For alternate hours, count whole rounds only until the next inlet interval can finish the tank.
- Treat the final interval separately.

Tank diagrams

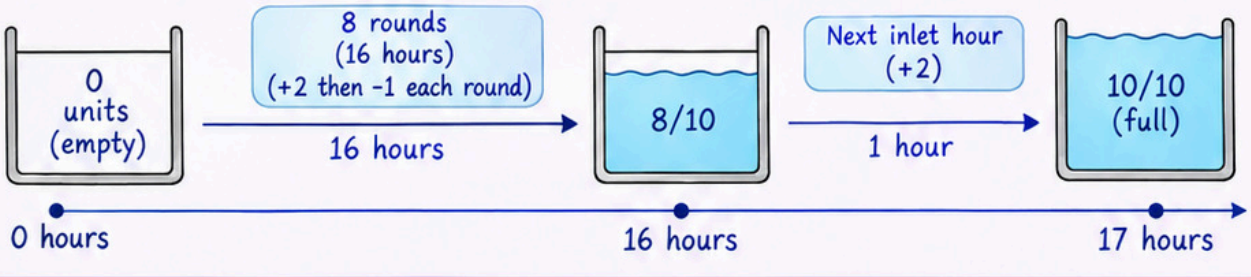


Type	Example	Working (LCM method)	Answer
A pipe closed after some time	An inlet fills in 6 hours and an outlet empties in 9 hours; both are opened on the empty tank, and the outlet is closed after 9 hours.	Capacity 18 units; net 3 − 2 = 1 unit an hour, so 9 units in 9 hours; the other 9 units at 3 units an hour take 3 hours.	Full 9 + 3 = 12 hours from the start.
Alternate hours	An inlet fills in 5 hours and an outlet empties in 10 hours; they are opened by turns for one hour each, the inlet first.	Capacity 10 units; +2 then −1, so 1 unit in each 2-hour round; after 8 rounds (16 hours) the tank holds 8 units, and the next inlet hour adds 2.	Full at 17 hours, not 20.

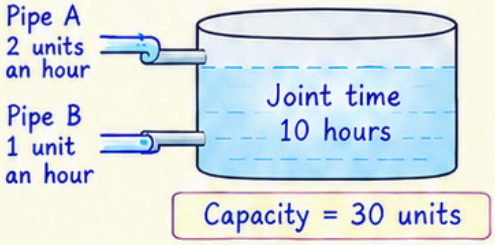
Timeline: a pipe closed after some time (6 hours and 9 hours)



Timeline: alternate hours (5 hours and 10 hours)



Efficiencies and the six traps

Type	Example	Working (LCM method)	Answer
Pipes of different efficiencies	Pipe A fills twice as fast as pipe B; together they fill the tank in 10 hours.	Rates 2 and 1, so 3 units an hour; capacity $3 \times 10 = 30$ units. 	A alone 15 hours; B alone 30 hours.

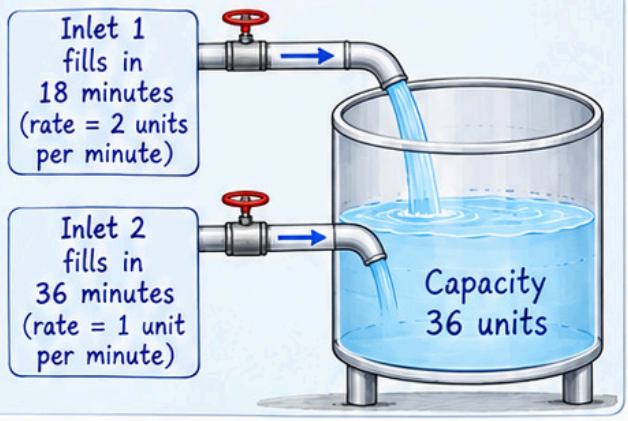
Remember the negative sign for every outlet.

Trap	What it looks like	The right way
1. Times added in place of rates	Inlets of 14 hours and 35 hours "take 49 hours".	Rates add: 10 hours.
2. Outlet counted as filling	Inlet 9 hours, outlet 15 hours: $5 + 3 = 8$ units an hour.	$5 - 3 = 2$ units an hour.
3. The net rate given as the answer	"2 units an hour", written as 2 hours.	Time = capacity ÷ net rate = $45 \div 2$ hours.
4. Work already done forgotten	After a pipe is closed, the whole capacity divided by the new rate.	Divide only what is left.
5. Whole rounds counted to the end	Alternate hours: 10 rounds of 2 hours = 20 hours.	The tank is full in the 17th hour.
6. The extra time read as the leak's time	A leak that adds $1\frac{1}{2}$ hours "empties the tank in $1\frac{1}{2}$ hours".	For an inlet filling in 6 hours, a leak delays filling to $7\frac{1}{2}$ hours. The leak alone takes 30 hours.

Practice drill 1-2

1. Two inlets. Two pipes fill a tank in 18 minutes and 36 minutes. How long do they take together?

Tank and inlet diagram



Solution

Capacity = 36 units.

Rates: 2 units per minute and 1 unit per minute.

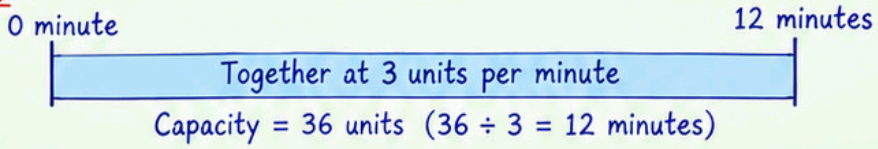
Together: 2 + 1 = 3 units per minute.

Time taken = 36 ÷ 3 = 12 minutes.

Formula:

$$18 \times 36 \div 54 = 12 \text{ minutes.}$$

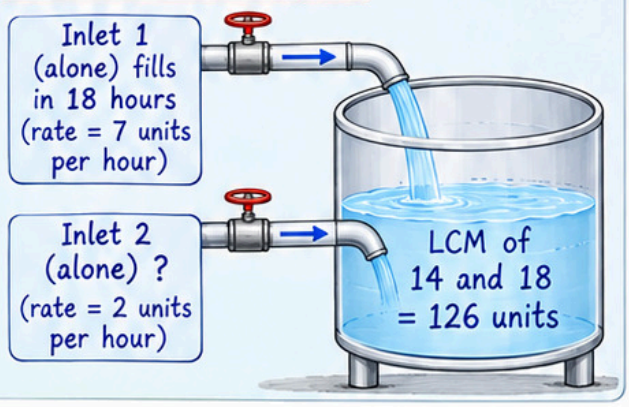
Timeline (filling)



Capacity = 36 units (36 ÷ 3 = 12 minutes)

2. One pipe from the pair. Two inlets together fill a tank in 14 hours, and the first alone in 18 hours. How long does the second take alone?

Tank and inlet diagram



Solution

The LCM of 14 and 18 is 126 units.

Together: 126 ÷ 14 = 9 units per hour.

First (alone): 126 ÷ 18 = 7 units per hour.

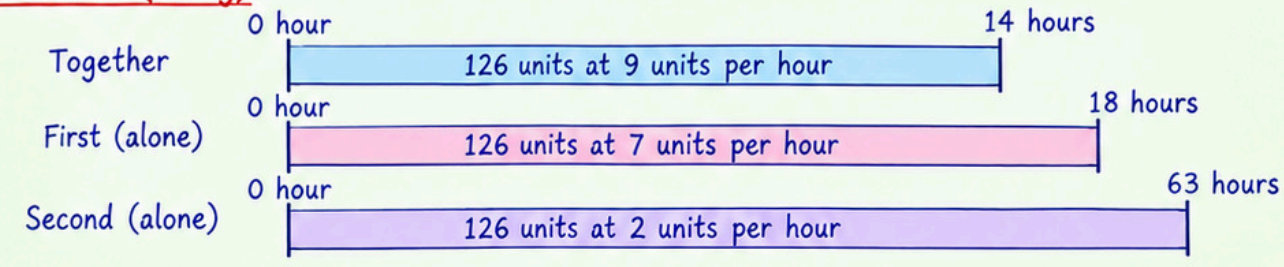
Second: 9 − 7 = 2 units per hour.

Time taken by the second = 126 ÷ 2 = 63 hours.

Formula:

$$18 \times 14 \div (18 - 14) = 63 \text{ hours.}$$

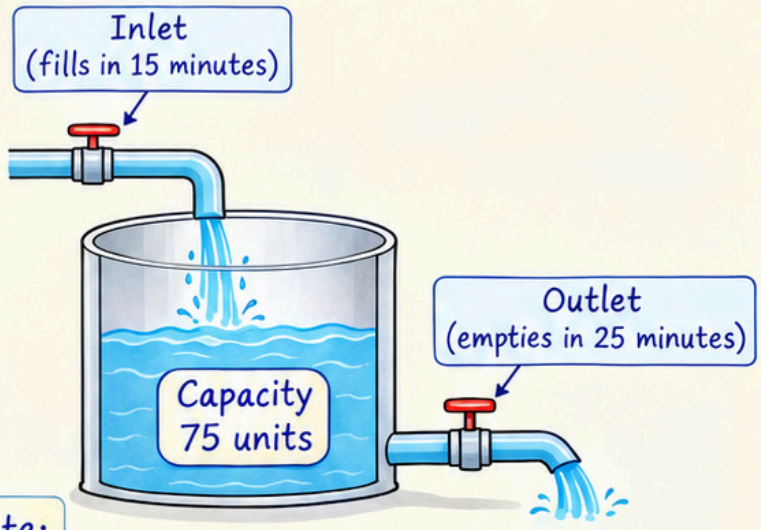
Timelines (filling)



Practice drill 3-4

3. Inlet and outlet.

An inlet fills a tank in 15 minutes and an outlet empties it in 25 minutes. Both are opened on the empty tank.



Capacity 75 units;

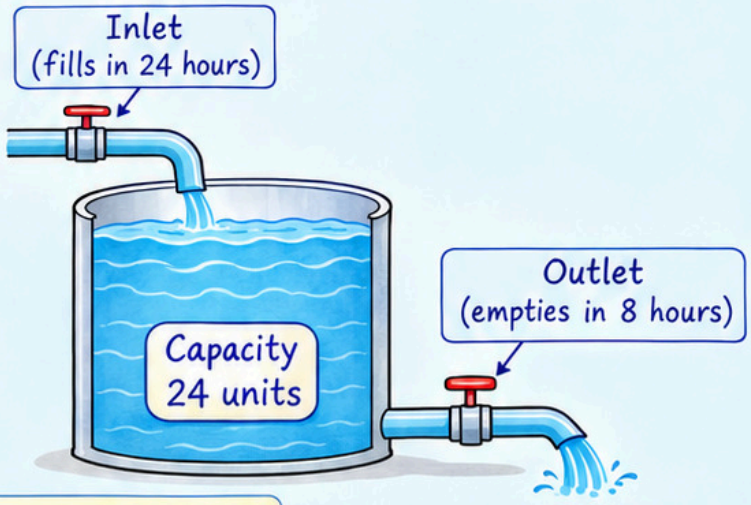
net $5 - 3 = 2$ units a minute;

$75 \div 2 = 37\frac{1}{2}$ minutes, that is, 37 minutes 30 seconds.

Formula: $15 \times 25 \div 10 = 37\frac{1}{2}$ minutes.

4. Emptying with the inlet on.

A full tank has an outlet that empties it in 8 hours and an inlet that fills it in 24 hours. Both are opened.



Capacity 24 units;

net $3 - 1 = 2$ units an hour going out;

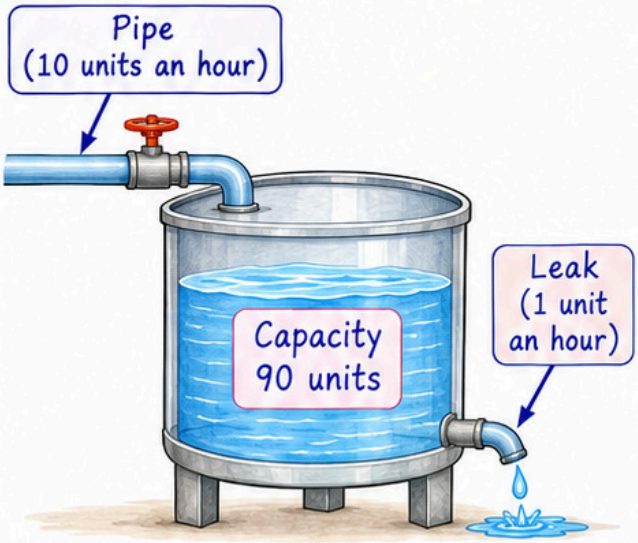
$24 \div 2 = 12$ hours to empty.

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Practice drill 5-6

5. A leak.

A pipe fills a tank in 9 hours, but because of a leak it takes 1 hour longer. How long does the leak alone take to empty the full tank?



Solution:

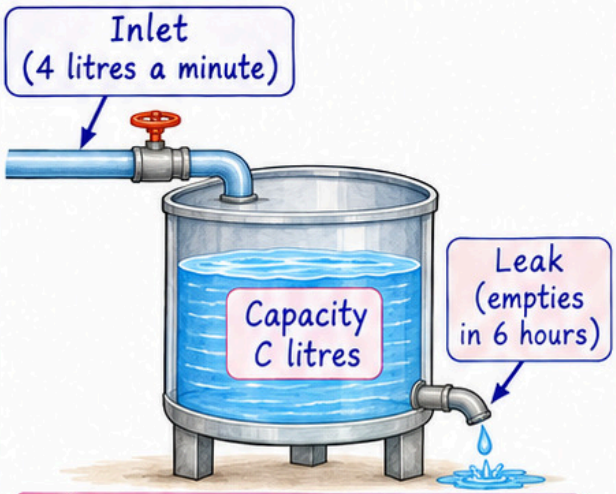
Capacity 90 units (the LCM of 9 and 10).

The pipe does 10 units an hour; with the leak only 9; so the leak takes away 1 unit an hour: 90 hours.

Formula: $9 \times 10 \div 1 = 90$ hours.

6. Capacity in litres.

A leak can empty a full tank in 6 hours. An inlet that lets in 4 litres a minute is opened, and the full tank now empties in 10 hours. What is the capacity?



Phases:
Leak alone: empties in 6 hours.
With inlet: empties in 10 hours.

Solution:

The inlet adds $4 \times 60 = 240$ litres an hour.

If the capacity is C litres, the leak takes out $\frac{C}{6}$ an hour, and $\frac{C}{6} - 240 = \frac{C}{10}$.

So $\frac{C}{15} = 240$ and $C = 3{,}600$ litres.

Check: the leak takes out 600 litres an hour;
$600 - 240 = 360$;
$3{,}600 \div 360 = 10$ hours.

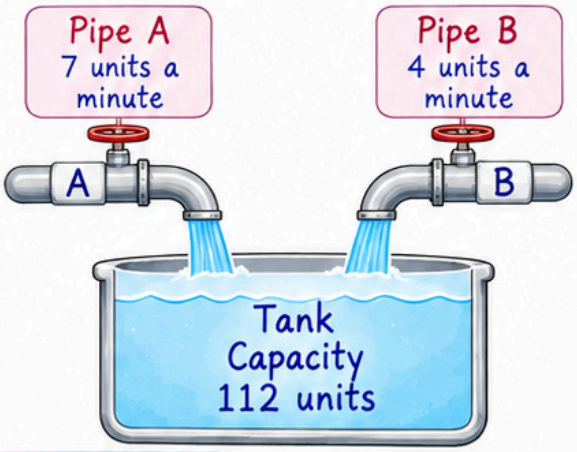
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Practice drill 7–8

7. A pipe opened later.

Pipe A fills a tank in 16 minutes and pipe B in 28 minutes. A is opened alone; B is opened 5 minutes later. When is the tank full?

Capacity 112 units; A does 7 and B 4 units a minute.



Working:

- In the first 5 minutes A fills 35 units.
- The other 77 units at 11 units a minute take 7 minutes.

Equations:

112 − 35 = 77 units.
7 + 4 = 11 units a minute.
77 ÷ 11 = 7 minutes.

Answer: Full 5 + 7 = 12 minutes after A was opened.

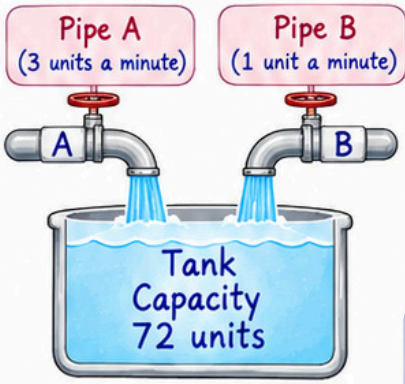
Phase diagram:

	A only (7 units a minute)	A and B together (11 units a minute)
Tank filling:		
Time:	← 5 minutes →	← 7 minutes →

Total time: 5 + 7 = 12 minutes.

8. Different efficiencies.

Pipe A fills a tank three times as fast as pipe B. Together they fill it in 18 minutes. How long does B take alone?



Working:

- Rates 3 and 1, so 4 units a minute; capacity 4 × 18 = 72 units.
- B alone takes 72 ÷ 1 = 72 minutes (A alone 24 minutes).
- Check: 24 × 72 ÷ 96 = 18 minutes.

Rates and time diagram:

Pipe	Rate (units a minute)	Time to fill alone
A	3	24 minutes
B	1	72 minutes
A and B	4	18 minutes

Equations:

4 × 18 = 72 units.
72 ÷ 1 = 72 minutes.
Check: 24 × 72 ÷ 96 = 18 minutes.

Answer:
B alone takes 72 minutes (A alone 24 minutes).

Three checks that prevent wrong answers

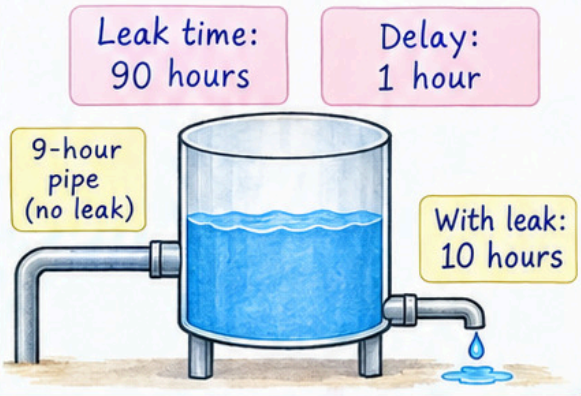
The three checks below help avoid common mistakes in pipe and tank questions. They show how to find the leak's time, how to handle alternate hours correctly, and how to decide which way the level moves.

The three checks:

1. The extra time is not the leak's time.
2. In alternate hours, the tank can be full before a round ends.
3. Which pipe is faster decides which way the level moves.

1. The extra time is not the leak's time.

If a leak makes a 9-hour pipe take 10 hours, the leak alone does not empty the tank in 1 hour. Its rate is the difference of the two rates, $\frac{1}{9} - \frac{1}{10} = \frac{1}{90}$, so it takes 90 hours: a slow leak makes only a small delay.



2. In alternate hours, the tank can be full before a round ends.

With an inlet that fills in 5 hours and an outlet that empties in 10 hours working by turns, the net gain is 1 unit in 2 hours out of 10 units, but the answer is not $10 \times 2 = 20$ hours. After 16 hours the tank holds 8 units, and the inlet's next hour fills the last 2 units, so it is full at 17 hours.

Alternate hours: full at 17 hours, not 20 hours

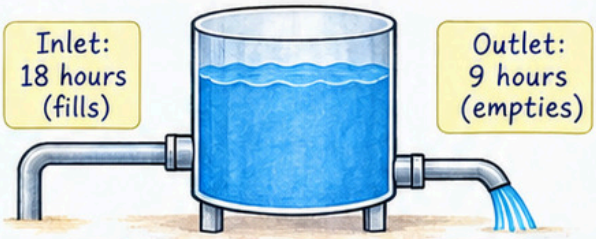
Time (hours)	Tank (units)
0	0
2	1
4	2
6	3
8	4
10	5
12	6
14	7
16	8
17	10



3. Which pipe is faster decides which way the level moves.

If the outlet empties faster than the inlet fills, an empty tank never fills, whatever the time. A full tank then empties at the outlet's rate minus the inlet's rate: an outlet of 9 hours against an inlet of 18 hours empties it in $9 \times 18 \div 9 = 18$ hours, not 9 hours.

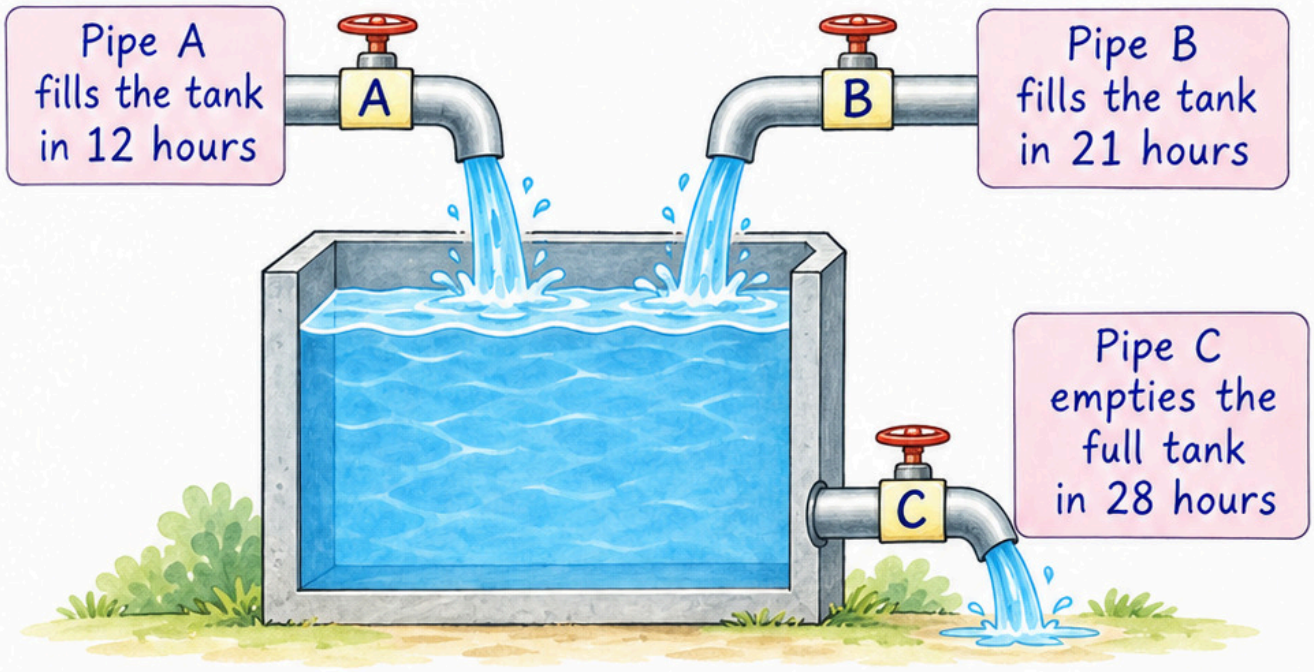
Faster outlet: empties in 18 hours



$9 \times 18 \div 9 = 18$ hours, not 9 hours.

Question 1

Q1. Two inlet pipes A and B can fill an empty tank in 12 hours and 21 hours respectively. An outlet pipe C can empty the full tank in 28 hours. If all three pipes are opened together when the tank is empty, in how much time will the tank be full?



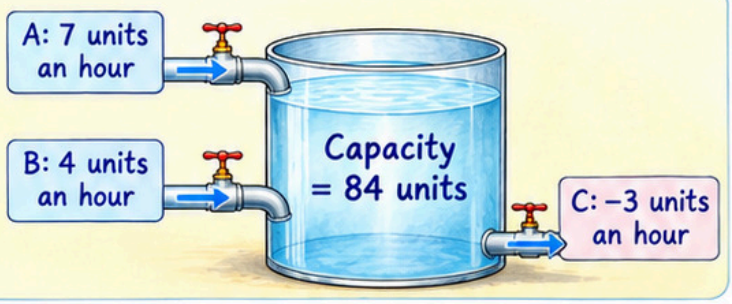
(1) 14 hours

(2) 12 hours

(3) 10 hours 50 minutes

(4) 10 hours 30 minutes

Question 1: worked solution



Worked solution

Step 1: $12 = 2^2 \times 3$, $21 = 3 \times 7$ and $28 = 2^2 \times 7$, so the LCM of 12, 21 and 28 is $2^2 \times 3 \times 7 = 84$. Take the capacity of the tank as 84 units.

Step 2: A fills $84 \div 12 = 7$ units an hour and B fills $84 \div 21 = 4$ units an hour. C empties $84 \div 28 = 3$ units an hour, so its rate is -3.

Step 3: net rate $= 7 + 4 - 3 = 8$ units an hour.

Step 4: time $= 84 \div 8 = 10\frac{1}{2}$ hours, that is, 10 hours 30 minutes.

Check by the rate method:

$$\frac{1}{12} + \frac{1}{21} - \frac{1}{28} = \frac{7}{84} + \frac{4}{84} - \frac{3}{84}$$

$$= \frac{8}{84} = \frac{2}{21} \text{ of the tank an hour,}$$

so the time is $\frac{21}{2}$ hours.

Rates and time summary

Pipe	Original time	Rate (units an hour)
A (inlet)	12 hours	7
B (inlet)	21 hours	4
C (outlet)	28 hours	−3
Together (net)	–	8
Capacity / Time	84 units	$10\frac{1}{2}$ hours

Sense check: A and B alone would take $84 \div 11 = 7\frac{7}{11}$ hours, and the outlet makes the time longer than that. B fills 4 units an hour and C empties only 3, so the three together fill faster than A alone, which takes 12 hours. The answer must lie between $7\frac{7}{11}$ hours and 12 hours. This rules out (1) and (2), but only the full working separates (3) from (4).

Why (1) is wrong: $14 = 84 \div (7 - 4 + 3)$. It puts the minus sign on the inlet B instead of the outlet C. Only the pipe that empties the tank gets the minus sign.

Why (2) is wrong: $12 = 84 \div 7$, the time of A alone. It treats B and C as if they cancel each other out. But B adds 4 units an hour and C takes away only 3, so the net rate is $7 + 4 - 3 = 8$, not 7.

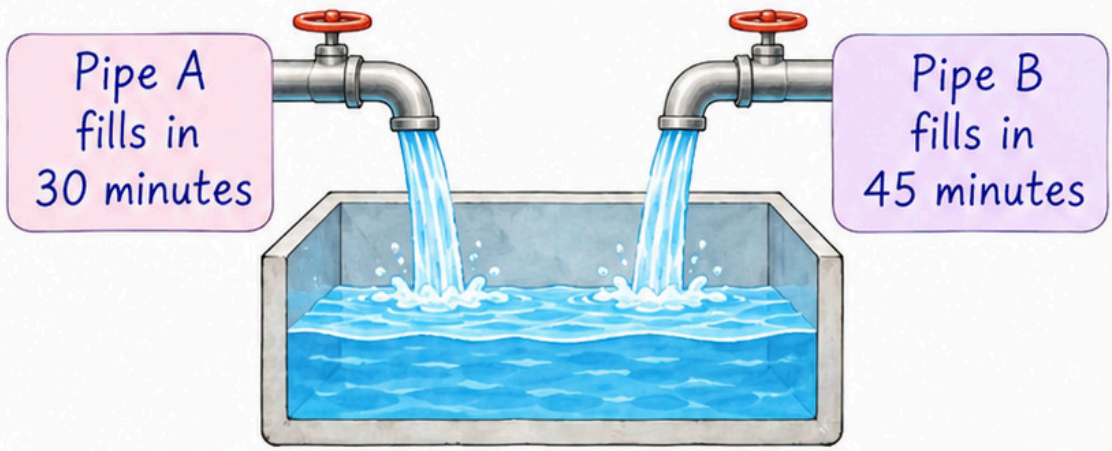
Why (3) is wrong: 10 hours 50 minutes reads 10.5 hours as 10 hours and 50 minutes. The time $84 \div 8 = 10\frac{1}{2}$ hours, and half an hour is 30 minutes, not 50.

Answer: (4) 10 hours 30 minutes

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Question 2

Q2. Pipes A and B can fill an empty tank in 30 minutes and 45 minutes respectively. Both pipes are opened together when the tank is empty. After 12 minutes pipe B is closed, and pipe A alone fills the rest of the tank. For how many more minutes after B is closed must A run to fill the tank?



Options:

(1) 10 minutes

(2) 15 minutes

(3) 22 minutes

(4) 30 minutes

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Question 2: worked solution

Answer: (1) 10 minutes

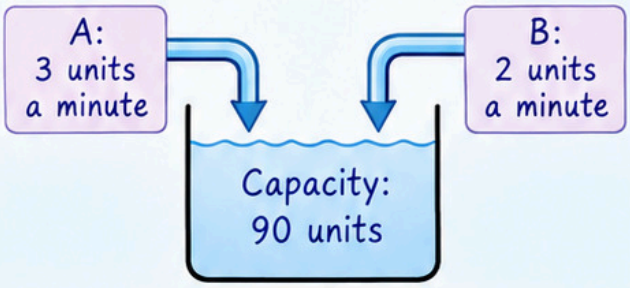
Worked solution

Step 1: the LCM of 30 and 45 is 90. Take the capacity as 90 units. A fills $90 \div 30 = 3$ units a minute and B fills $90 \div 45 = 2$ units a minute.

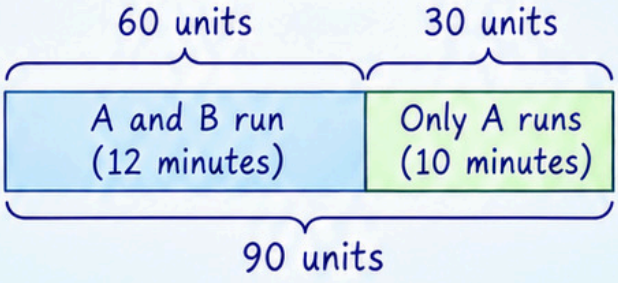
Step 2: in the first 12 minutes both pipes run: $12 \times (3 + 2) = 60$ units.

Step 3: $90 - 60 = 30$ units are left, and only A runs now: $30 \div 3 = 10$ more minutes.

Capacity / rate diagram



Progress diagram (units)



Check by the rate method: in 12 minutes the two fill

$$12 \times \left(\frac{1}{30} + \frac{1}{45}\right) = 12 \times \frac{5}{90} = \frac{2}{3} \text{ of the tank.}$$

$\frac{1}{3}$ is left, and A takes $\frac{1}{3} \div \frac{1}{30} = 10$ minutes.

Why (2) is wrong: $15 = 30 \div 2$. It divides the units left by B's rate, but B has been closed; only A fills the rest.

Why (3) is wrong: $22 = 12 + 10$ is the time from the start. The question asks for the minutes after B is closed.

Why (4) is wrong: $30 = 90 \div 3$ is the time A takes to fill the whole tank alone. It forgets the 60 units already filled in the first 12 minutes.

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