CSIR: organisation and purpose

The Council of Scientific and Industrial Research (CSIR) is a registered society under the Department of Scientific and Industrial Research (DSIR), Ministry of Science and Technology. It operates a national network of research laboratories.

Key institutional details

- **Name:** Council of Scientific and Industrial Research (CSIR)
- **Legal status:** Registered society
- **Under:** Department of Scientific and Industrial Research (DSIR)
- **Ministry:** Ministry of Science and Technology
- **Network:** Runs a network of national laboratories spread across India

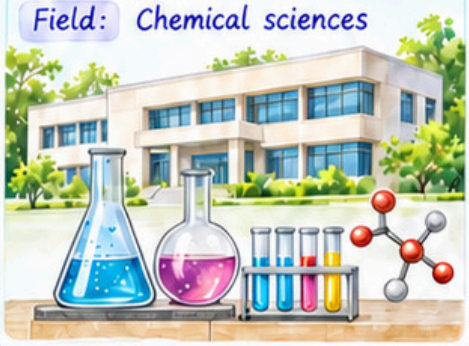


CSIR at a glance

- Origin and legal status
- Leadership
- Laboratories and cities
- Research fields
- Missions
- Awards

Network of national laboratories (illustrative)

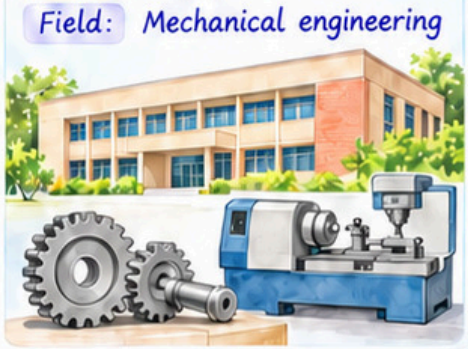
Laboratory: CSIR-NCL
City: Pune
Field: Chemical sciences



Laboratory: CSIR-IITR
City: Lucknow
Field: Toxicology



Laboratory: CSIR-CMERI
City: Durgapur
Field: Mechanical engineering



Missions



Awards



Illustrations are schematic

2.1 What CSIR is

Point	Fact (as on 7 November 2026)
Full name	Council of Scientific and Industrial Research
Origin	Registered as a society on 12 March 1942; the Government Resolution on CSIR is dated 26 September 1942
Legal status	A society registered under the Societies Registration Act (XXI of 1860)
President	The Prime Minister, ex officio (Shri Narendra Modi)
Vice-President	The Minister in charge of the ministry or department that deals with CSIR, ex officio (Dr Jitendra Singh, Minister of State for Science and Technology and Earth Sciences)
Director-General	The principal executive officer of the society; Dr (Smt) N. Kalaiselvi, Director-General, CSIR and Secretary, DSIR
Parent department	DSIR, Ministry of Science and Technology
Headquarters	Anusandhan Bhawan, 2 Rafi Ahmed Kidwai Marg, New Delhi
Network	37 national laboratories, 39 outreach centres, 1 innovation complex and 3 units with pan-India presence

- "Ex officio" means by virtue of the office: whoever holds the office holds the CSIR post.

- CSIR is under DSIR, not the Department of Science and Technology (DST).



President
(The Prime Minister)
(Shri Narendra Modi)

↓

Vice-President
(The Minister in charge of the ministry or department that deals with CSIR)
(Dr Jitendra Singh, Minister of State for Science and Technology and Earth Sciences)

↓

Director-General
(The principal executive officer of the society)
(Dr (Smt) N. Kalaiselvi)



37	39	1	3
national laboratories	outreach centres	innovation complex	units with pan-India presence

Illustrations are schematic

2.2 Origin and governance

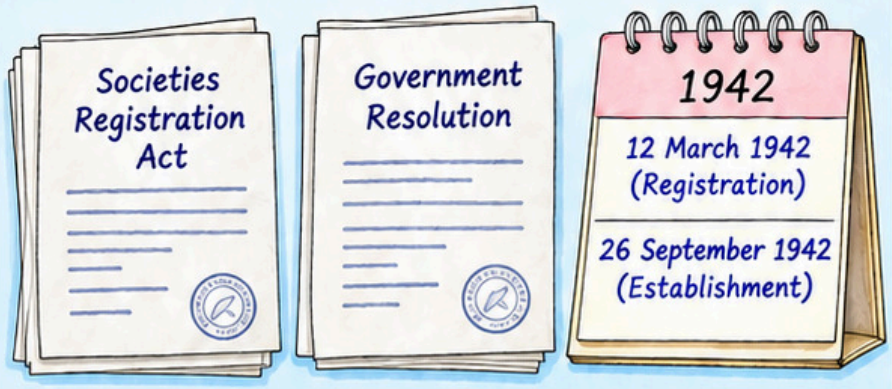
1 April 1940: Board of Scientific and Industrial Research (BSIR) created; Shanti Swarup Bhatnagar appointed Director. He became CSIR's founder Director.



12 March 1942: CSIR registered under the Societies Registration Act.

26 September 1942: Government Resolution establishing CSIR.

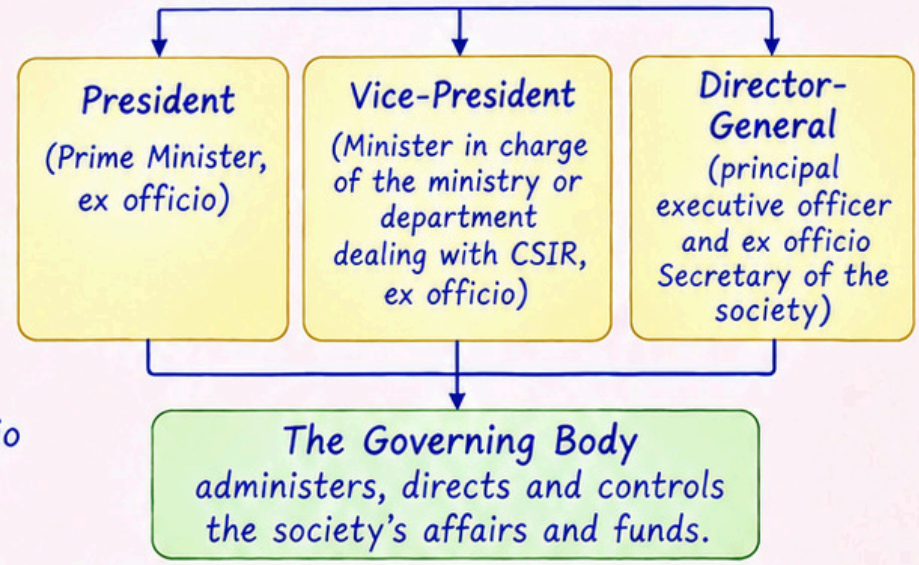
Registration and establishment are distinct dates.



President: Prime Minister, ex officio.

Vice-President: Minister in charge of the ministry or department dealing with CSIR, ex officio.

Director-General: principal executive officer and ex officio Secretary of the society.



N. Kalaiselvi became Director-General in August 2022. As on 7 November 2026, she is Director-General, CSIR and Secretary, DSIR.

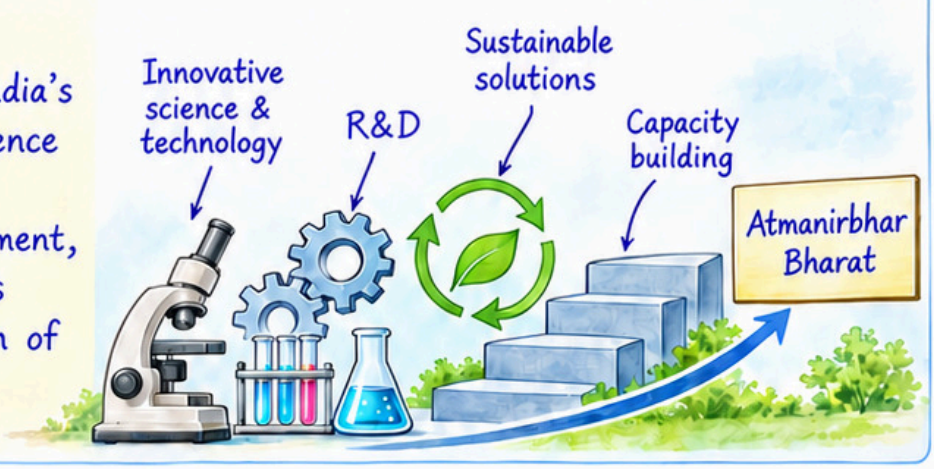


Illustrations are schematic

Vision and mission

Vision:

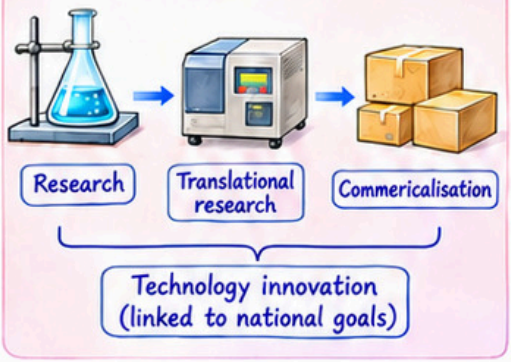
to raise the quality of life of India's citizens through innovative science and technology and globally competitive research and development, by building sustainable solutions and capacity to fulfil the dream of Atmanirbhar Bharat.



Mission:

thirteen points. Among them:

- technology innovation, translational research and commercialisation linked to national goals;



- design and development of small and medium civil aircraft;



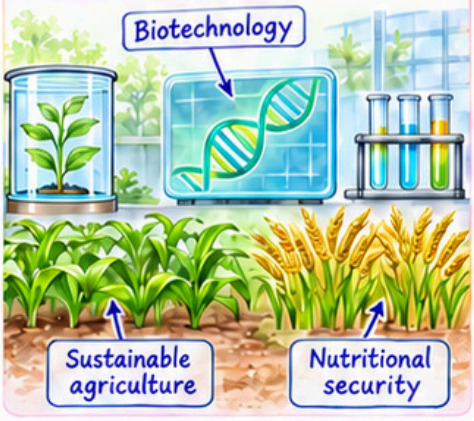
- scalable green energy technologies;



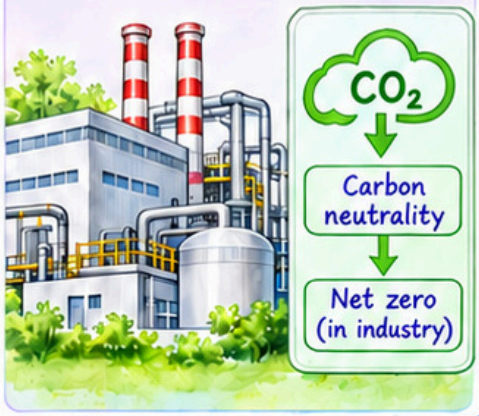
- globally benchmarked sustainable chemical processes that reduce the trade deficit;



- sustainable agriculture and nutritional security through biotechnology;



- technologies for carbon neutrality and net zero in industry.



Illustrations are schematic

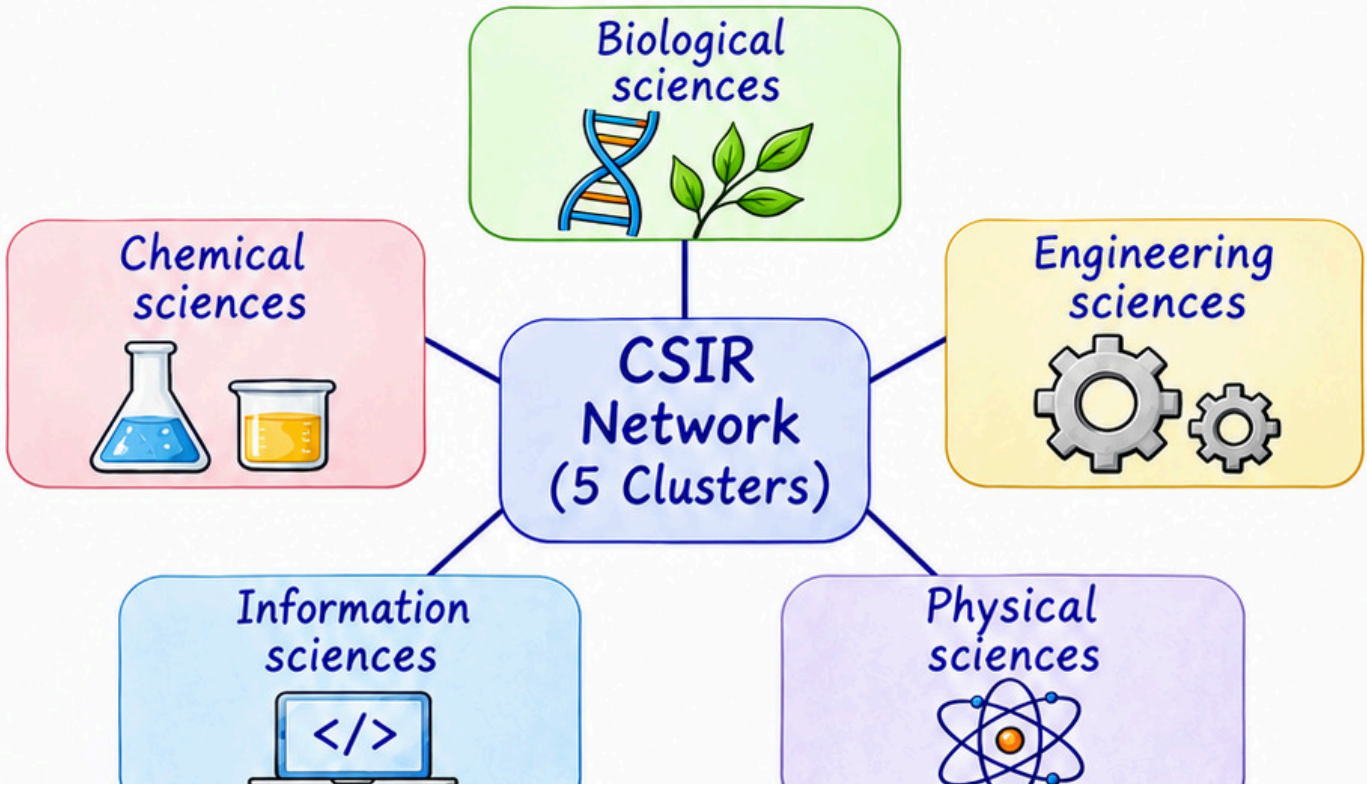
2.4 The size of the network (as on 7 November 2026)

- As on 7 November 2026: 37 national laboratories, 39 outreach centres, 1 innovation complex and 3 units.
- CSIR-Innovation Complex is in Mumbai.
- AcSIR, Ghaziabad; TKDL, New Delhi; and URDIP, Pune are other bodies/units, not members of the 37-laboratory count.



National laboratories	= 37
Outreach centres	= 39
Innovation complex	= 1
Units	= three

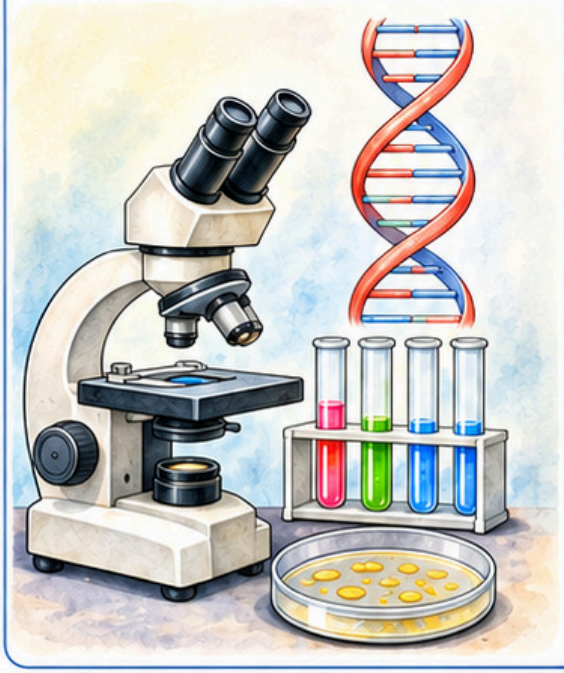
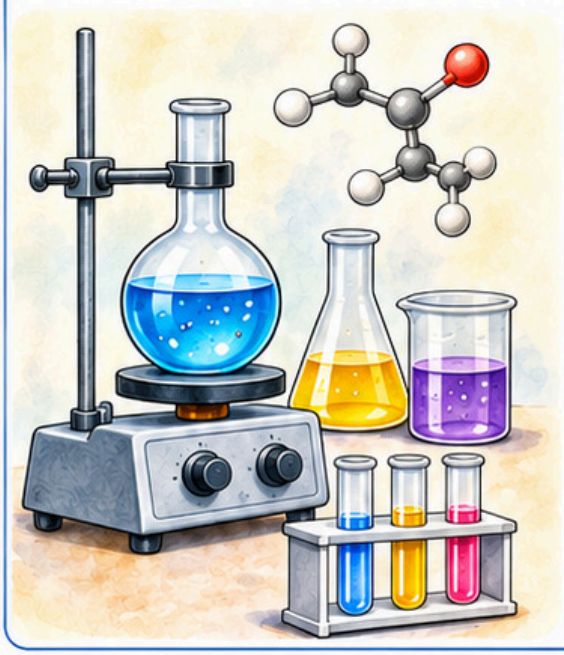
Five clusters: biological sciences, chemical sciences, engineering sciences, information sciences and physical sciences.



Illustrations are schematic

2.5 The laboratories by field

The 37 laboratories are grouped into five scientific clusters. AcSIR, TKDL and URDIP are not included in this laboratory count.

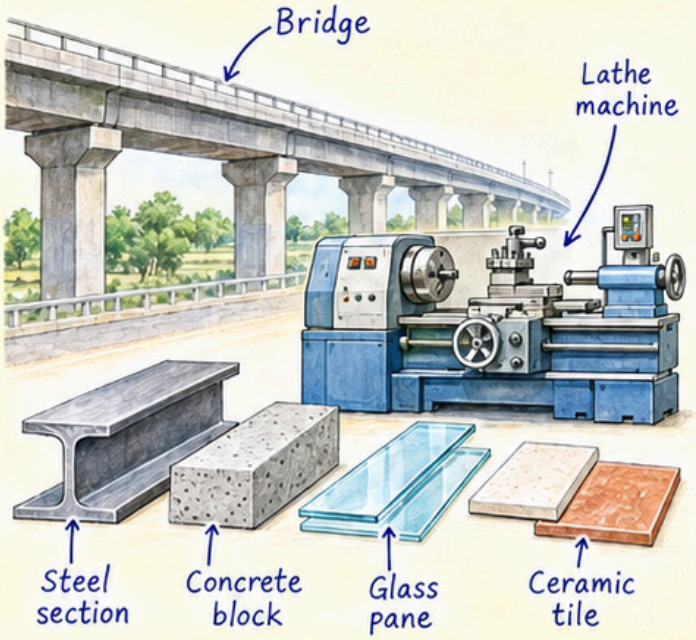
CSIR cluster (number of laboratories)	Laboratories and their cities
Biological sciences (11) 	• Centre for Cellular and Molecular Biology (CCMB), Hyderabad; • Central Drug Research Institute (CDRI), Lucknow; • Central Institute of Medicinal and Aromatic Plants (CIMAP), Lucknow; • Central Food Technological Research Institute (CFTRI), Mysuru; • Institute of Genomics and Integrative Biology (IGIB), Delhi; • Institute of Himalayan Bioresource Technology (IHBT), Palampur; • Indian Institute of Chemical Biology (IICB), Kolkata; • Indian Institute of Integrative Medicine (IIIM), Jammu; • Institute of Microbial Technology (IMTECH), Chandigarh; • Indian Institute of Toxicology Research (IITR), Lucknow; • National Botanical Research Institute (NBRI), Lucknow.
Chemical sciences (9) 	• Central Electrochemical Research Institute (CECRI), Karaikudi; • Central Leather Research Institute (CLRI), Chennai; • Central Institute of Mining and Fuel Research (CIMFR), Dhanbad; • Central Salt and Marine Chemicals Research Institute (CSMCRI), Bhavnagar; • Indian Institute of Chemical Technology (IICT), Hyderabad; • Indian Institute of Petroleum (IIP), Dehradun; • National Chemical Laboratory (NCL), Pune; • North-East Institute of Science and Technology (NEIST), Jorhat; • National Institute for Interdisciplinary Science and Technology (NIIST), Thiruvananthapuram.

Illustrations are schematic

2.5 The laboratories by field (2/2)

CSIR cluster (number of laboratories)	Laboratories and their cities

Engineering sciences (10)



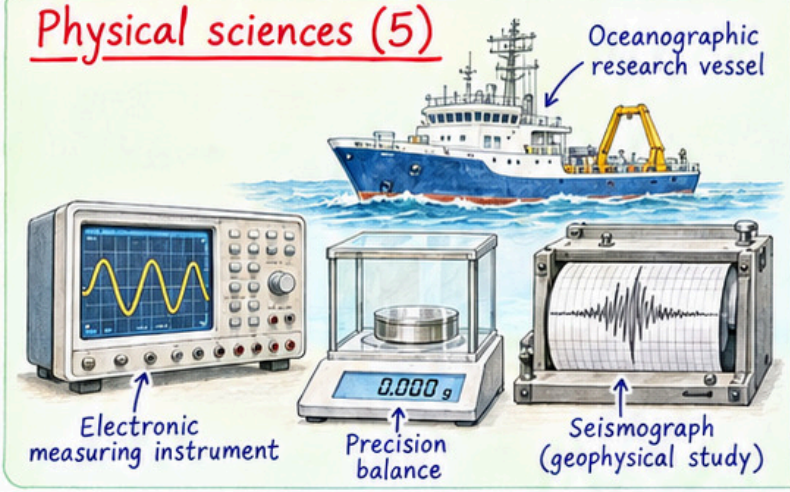
- Advanced Materials and Processes Research Institute (AMPRI), Bhopal;
- Central Building Research Institute (CBRI), Roorkee;
- Central Glass and Ceramic Research Institute (CGCRI), Kolkata;
- Central Mechanical Engineering Research Institute (CMERI), Durgapur;
- Central Road Research Institute (CRRI), New Delhi;
- Institute of Minerals and Materials Technology (IMMT), Bhubaneswar;
- National Aerospace Laboratories (NAL), Bengaluru;
- National Environmental Engineering Research Institute (NEERI), Nagpur;
- National Metallurgical Laboratory (NML), Jamshedpur;
- Structural Engineering Research Centre (SERC), Chennai

Information sciences (2)



- National Institute of Science Communication and Policy Research (NISCPR), New Delhi;
- National Institute of Data Science and Artificial Intelligence (formerly CSIR-4PI), Bengaluru

Physical sciences (5)



- Central Electronics Engineering Research Institute (CEERI), Pilani;
- Central Scientific Instruments Organisation (CSIO), Chandigarh;
- National Geophysical Research Institute (NGRI), Hyderabad;
- National Institute of Oceanography (NIO), Goa;
- National Physical Laboratory (NPL), New Delhi

Illustrations are schematic

Laboratory–city pairs

Several laboratories in one city: Hyderabad has three (CCMB, IICT, NGRI). Lucknow has four (CDRI, CIMAP, NBRI, IITR). Chandigarh has two: CSIO, the instruments laboratory, and IMTECH, the microbial technology laboratory. Chennai has two (CLRI and SERC), Kolkata two (IICB and CGCRI), and New Delhi three (NPL, CRRI and NISCPR).

Mining and mechanical engineering:

the mining and fuel laboratory (CIMFR) is at Dhanbad; the mechanical engineering laboratory (CMERI) is at Durgapur; the metallurgy laboratory (NML) is at Jamshedpur.



NPL, New Delhi:

is the National Metrology Institute (NMI) of India. Its mandate is to develop measurement and calibration standards that are internationally accepted. CSIR-NPL maintains Indian Standard Time (IST).







Illustrations are schematic

2.7 Andhra Pradesh and Hyderabad

Institutional comparison

CSIR laboratories in Hyderabad

CCMB (cellular and molecular biology)
IICT (chemical technology)
NGRI (geophysics)

Hyderabad is in Telangana.

DRDO laboratories in Hyderabad

DRDL, RCI, ASL and DMRL.

These belong to the Ministry of Defence, not to CSIR.

Andhra Pradesh

None of the 37 laboratories in the CSIR network is located in Andhra Pradesh (as on 7 November 2026).

2.8 Missions and achievements

Mission or achievement refers to a focused government or scientific initiative that aims to promote a specific activity, expand its scale and create wider benefits, such as in agriculture and aroma crops.

Key takeaways:

- Aroma Mission has been implemented in two phases.
- It promotes cultivation of aromatic plants, builds skilled human resources and supports farming families.
- Lavender in Jammu and Kashmir has expanded with planting support and distillation units.
- Bhaderwah is recognised as the Lavender capital of India.

Aroma and lavender value chain (illustration)



Mission or achievement	What it is	Status date
Aroma Mission, Phase I	Launched by the Union Ministry of Science and Technology. In Phase I, CSIR helped cultivation on 6,000 hectares, covered 46 Aspirational districts and trained more than 44,000 persons	As of 16 January 2022
Aroma Mission, Phase II	Launched after Phase I. It proposed to engage over 45,000 skilled human resources and benefit more than 75,000 farming families. CSIR-IHBT, CSIR-CIMAP, CSIR-NBRI and CSIR-NEIST take part in Phase II along with CSIR-IIIM	As of 16 January 2022
Lavender in Jammu and Kashmir	CSIR-IIIM, Jammu, introduced lavender (its variety RRL-12), gave more than 30 lakh free lavender plants and installed 50 distillation units (45 fixed and 5 mobile). More than 2,500 farmers grow lavender in Jammu and Kashmir. This is known as the "Purple Revolution"	As of 4 June 2023
Bhaderwah	Bhaderwah is known as the Lavender capital of India. Lavender oil output there rose from 300 litres in 2019 to 1,500 litres in 2022	As of 4 June 2023

Illustrations are schematic

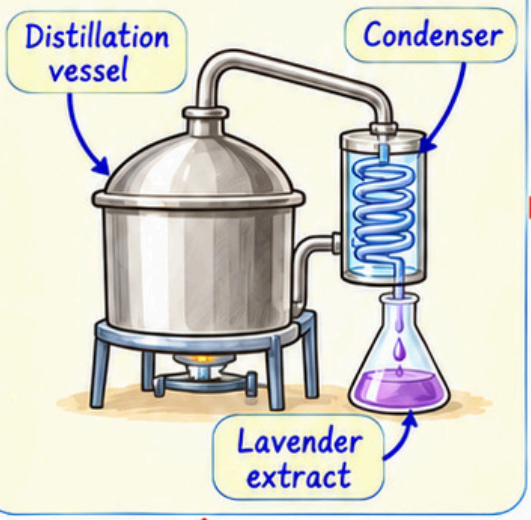
2.8 Missions and achievements

Mission or achievement	What it is	Status date
Rural development award	CSIR-IIIM received the CSIR award for S&T innovations for rural development (CAIRD-2020) for its lavender work	As of 4 June 2023

Lavender cultivation



Distillation unit



Lavender oil





Higher income for small and marginal farmers



Agriculture-based start-ups

- The Aroma Mission is a flagship project of CSIR under which lavender cultivation is promoted. Its aim is to raise the income of small and marginal farmers and to develop agriculture-based start-ups.

Illustrations are schematic

Science awards

The Shanti Swarup Bhatnagar Prize is named after CSIR's founder Director. It was first awarded in 1958 and covered seven fields of science.

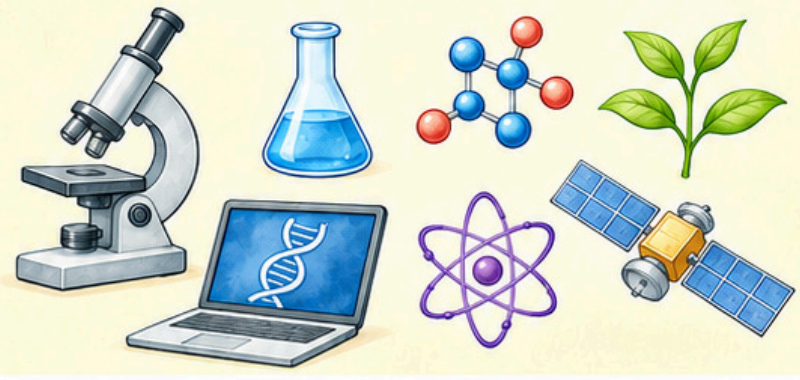
Rashtriya Vigyan Puraskar was announced on 21 September 2023.

Rashtriya Vigyan Puraskar

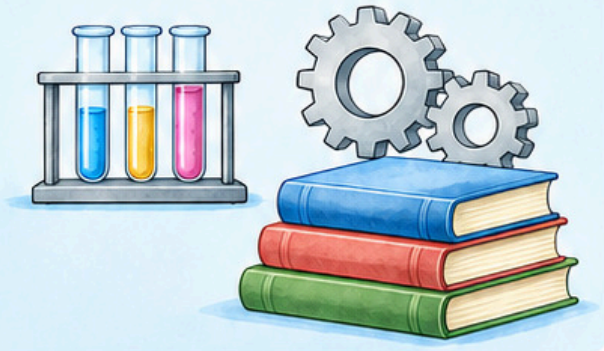
- Vigyan Ratna
- Vigyan Shri
- Vigyan Yuva-Shanti Swarup Bhatnagar (for young scientists up to 45 years
- Vigyan Team

Four categories across 13 domains.

Seven fields of science



13 domains



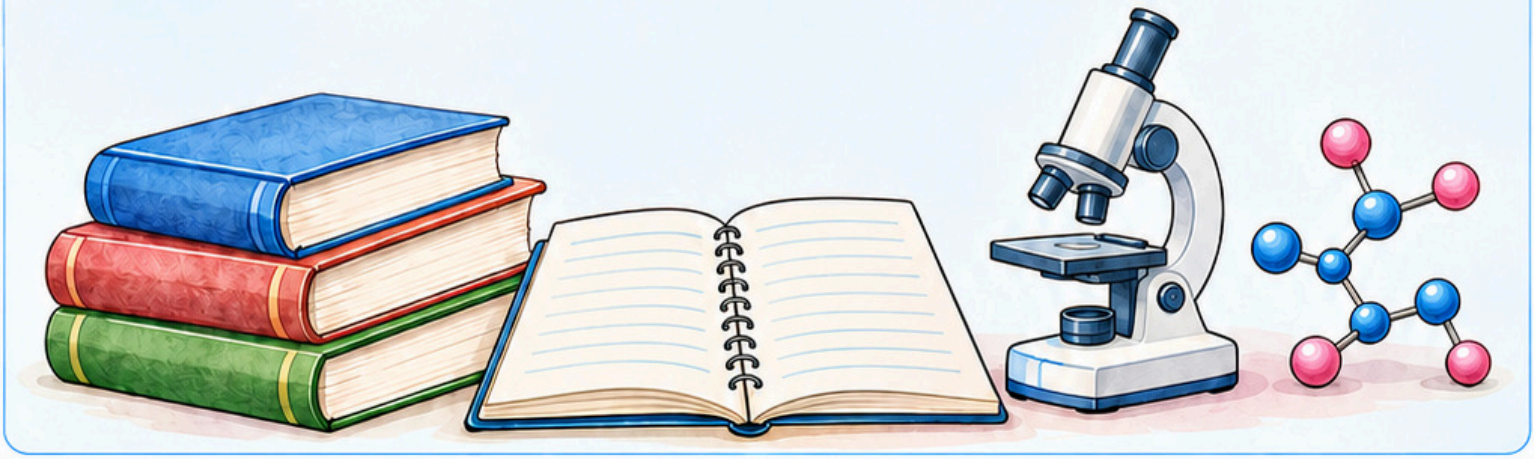
Illustrations are schematic

CSIR and other research organisations

- DRDO is the research and development wing of the Ministry of Defence, formed in 1958. As on 17 October 2026, it has around 41 laboratories and 5 Young Scientist Laboratories.
- Keep CSIR's laboratory network separate from DRDO and from research bodies such as ICAR, ICMR, DBT, IISc, IITs and the Anusandhan National Research Foundation.

Indian Research Organisations

- **CSIR** — laboratory network
- **DRDO**
- ICAR, ICMR and DBT
- IISc, the IITs and other national research institutes
- **Anusandhan** National Research Foundation

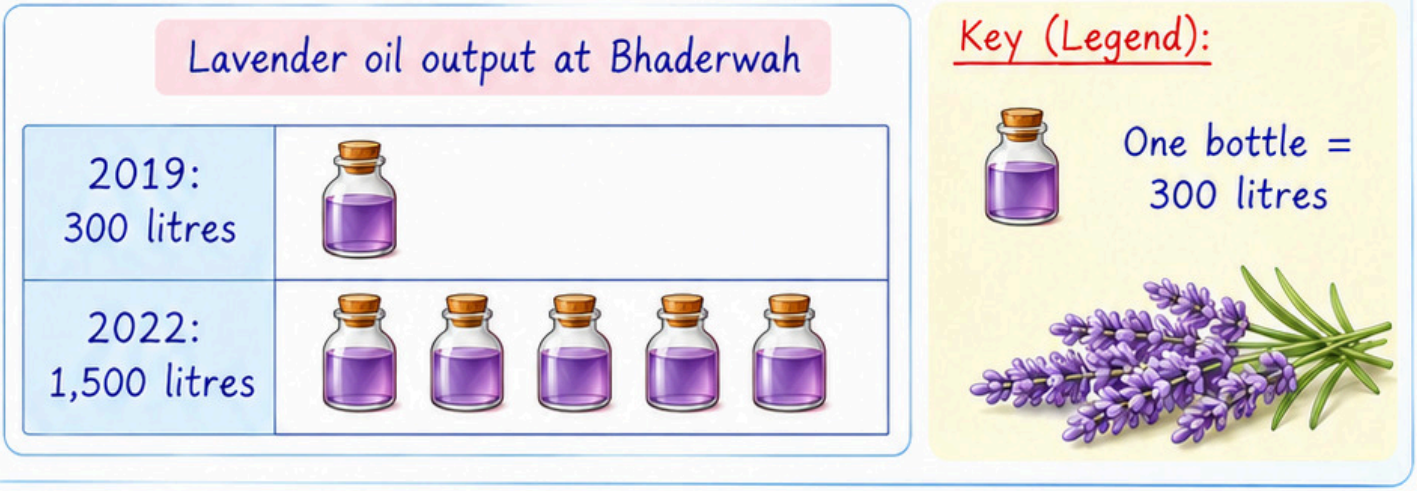


Illustrations are schematic

Extra section: CSIR in date order

Registration: 12 March 1942 → **Establishment:** 26 September 1942

Example: Pictograph (Lavender oil output at Bhaderwah)



CSIR in date order

Date	Event
1 April 1940	Board of Scientific and Industrial Research created, with Shanti Swarup Bhatnagar as Director
12 March 1942	CSIR registered as a society under the Societies Registration Act (XXI of 1860)
26 September 1942	Government Resolution on CSIR; CSIR established
1958	Shanti Swarup Bhatnagar Prize first given by CSIR
2019 to 2022	Lavender oil output at Bhaderwah rises from 300 litres to 1,500 litres
By 16 January 2022	Aroma Mission Phase I completed (6,000 hectares, 46 Aspirational districts); Phase II launched
August 2022	N. Kalaiselvi becomes Director-General of CSIR
4 June 2023	Bhaderwah called the "Lavender capital of India"
21 September 2023	Rashtriya Vigyan Puraskar announced, with a Vigyan Yuva-Shanti Swarup Bhatnagar category
7 November 2026	CSIR gives 37 national laboratories and 39 outreach centres

Key distinctions

1. Counting.

CSIR gives 37 national laboratories and 39 outreach centres (as on 7 November 2026). Do not swap the two figures.

DRDO's "around 41 laboratories" belongs to a different organisation, under the Ministry of Defence.





2. Look-alike "National" laboratories.

NAL is the aerospace laboratory at Bengaluru; NML is the metallurgy laboratory at Jamshedpur; NCL is the chemistry laboratory at Pune; NBRI is the botanical laboratory at Lucknow; NEERI is the environmental engineering laboratory at Nagpur.

One changed letter moves a laboratory to another city and another field.











3. CSIR or DRDO in Hyderabad.

CCMB, IICT and NGRI are CSIR laboratories, under DSIR in the Ministry of Science and Technology.

DRDL, RCI, ASL and DMRL are DRDO laboratories, under the Ministry of Defence.

A question that puts IICT under DRDO, or DRDL under CSIR, is wrong.





Illustrations are schematic

Numerical Venn diagrams

A numerical Venn diagram question gives the number of members in two or three groups and in some of their overlaps, and asks for the number in one region, or for the least or greatest possible overlap.

Solving method:

- Use the inclusion-exclusion formula.
- Fill the Venn diagram from the centre outwards.
- Use two limits on the overlap of two groups.

Inclusion-exclusion formula

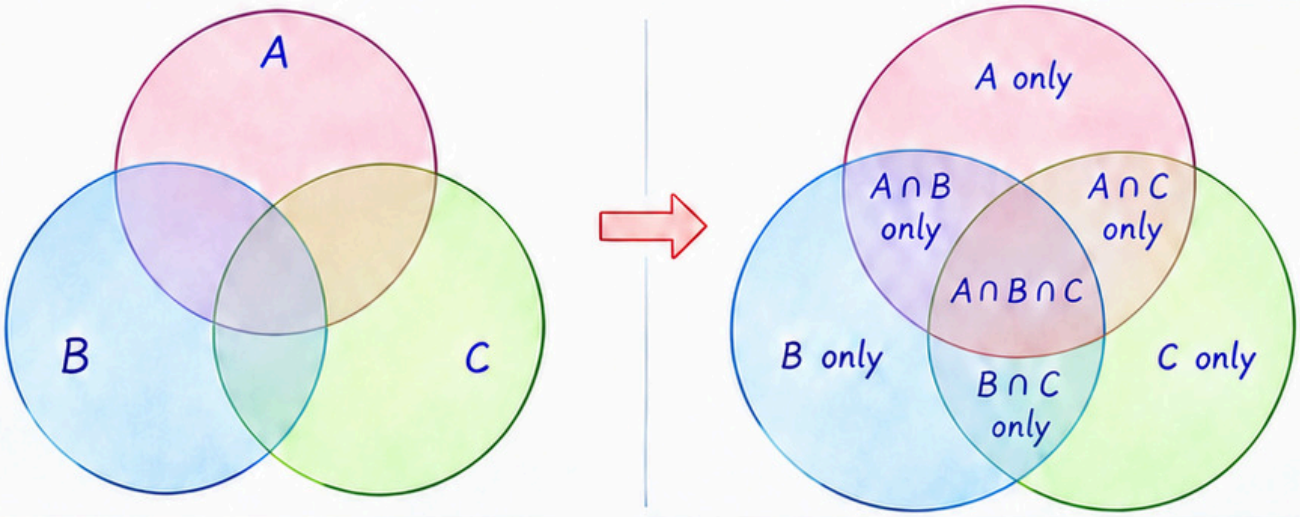
For two groups A and B:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

For three groups A, B and C:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Filling the Venn diagram (from the centre outwards)



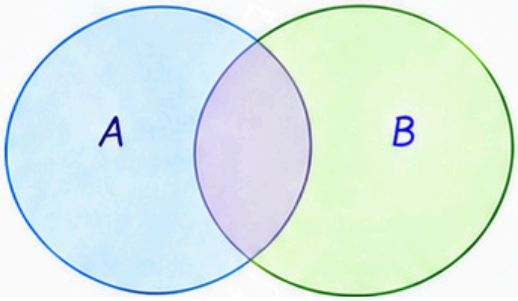
Two limits on the overlap of two groups

For two groups A and B:

Least possible overlap:
$n(A \cap B) = \max\{0, n(A) + n(B) - n(U)\}$
$n(U)$ = total people counted

Greatest possible overlap:
$n(A \cap B) \leq \min\{n(A), n(B)\}$

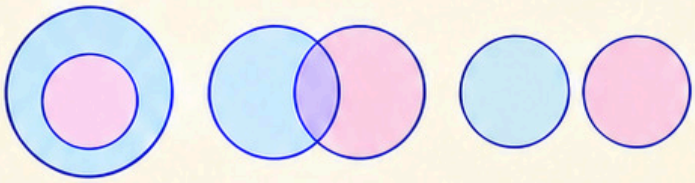
Two-group overlap



Venn foundations

Relationships between groups

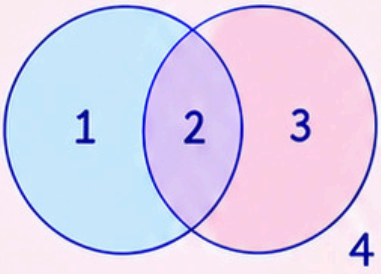
Circles show how groups are related (one inside another, partly overlapping, or separate); the size of a circle says nothing about how many members the group has.



Regions and counting

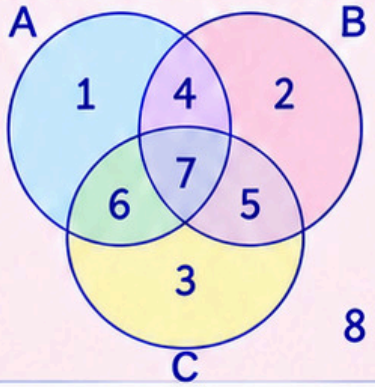
Two overlapping circles make 4 regions and three make 8; each phrase ("only A", "exactly two", "at least one", "none") names fixed regions; the two- and three-set formulas; filling a diagram from the eentre.
Use the same region numbers consistently.

Two sets: 4 regions



1: only A
2: both A and B
3: only B
4: none

Three sets: 8 regions



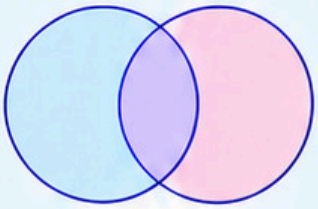
1: only A
2: only B
3: only C
4: A and B but not C
5: B and C but not A
6: C and A but not B
7: All three
8: None

Percentages

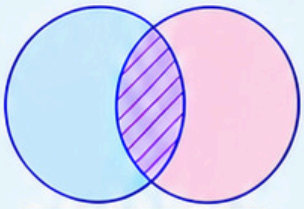
A percentage is a fraction with denominator 100; $x\%\ \text{of}\ y = \left(\frac{x}{100}\right) \times y$..

Working method

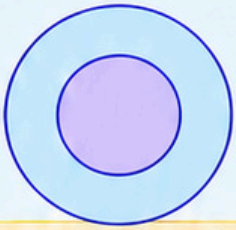
Why each step of the method works, how to find the centre when it is not given, how to check that the data are possible, the least and greatest overlap of two groups, and percentage versions.



centre



least overlap



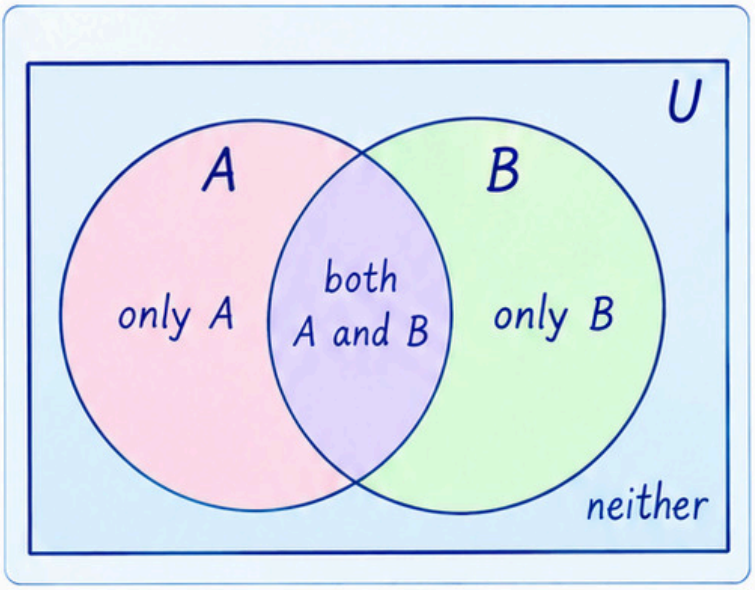
greatest overlap

$$x\%\ \text{of}\ y = \left(\frac{x}{100}\right) \times y$$

percentage version

2.2 Two groups: the formula, "only" and "neither" (1/2)

The diagram in words. A rectangle stands for everyone counted, the universal set U, with $n(U)$ members. Inside it, two circles A and B partly overlap. They make four regions: only A (inside A, outside B), both A and B (the common part), only B, and neither (inside the rectangle, outside both circles). Write $n(X)$ for the number of members of X.



Union:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Why: adding $n(A)$ and $n(B)$ counts each member of the common part twice, once in each circle. Subtracting $n(A \cap B)$ once leaves every member counted once.

Only A

$$\text{Only } A = n(A) - n(A \cap B),$$

because circle A is made of "only A" and the common part. In the same way,

$$\text{only } B = n(B) - n(A \cap B).$$

Neither

$$\text{Neither} = n(U) - n(A \cup B),$$

because the four regions together fill the rectangle.

Working backwards.

The five numbers $n(U)$, $n(A)$, $n(B)$, $n(A \cap B)$ and "neither" are tied by one equation:

$$n(U) = n(A) + n(B) - n(A \cap B) + \text{neither}.$$

Any four of them give the fifth.

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2.2 Two groups: the formula, "only" and "neither" (2/2)

For two groups, we can find the number who have at least one, neither and only one of the two, using simple formulas.

Key idea:

Using a Venn diagram for two groups, the overlap is "both", the two non-overlapping parts are "only" each group, and the area outside both is "neither".

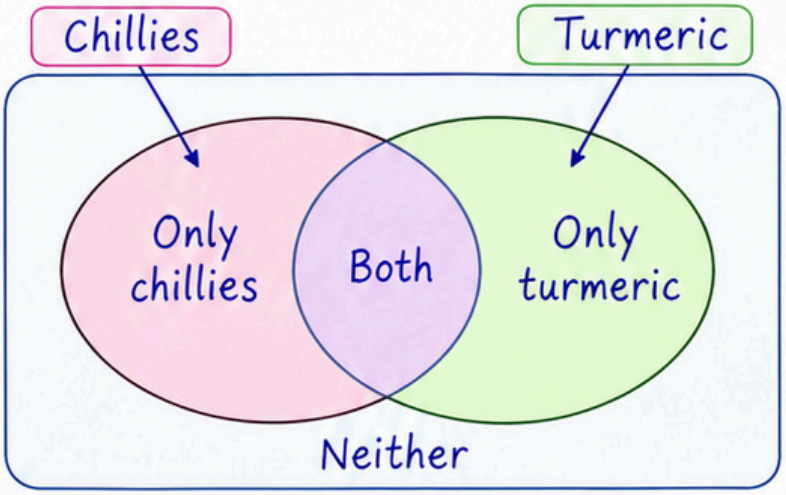
Formulas for two groups:

- At least one = (first group) + (second group) − (both)
- Neither = (total) − (at least one)
- Only first group = (first group) − (both)
- Only second group = (second group) − (both)

Example:

Example: of 75 farmers, 48 grow chillies, 39 grow turmeric and 21 grow both.



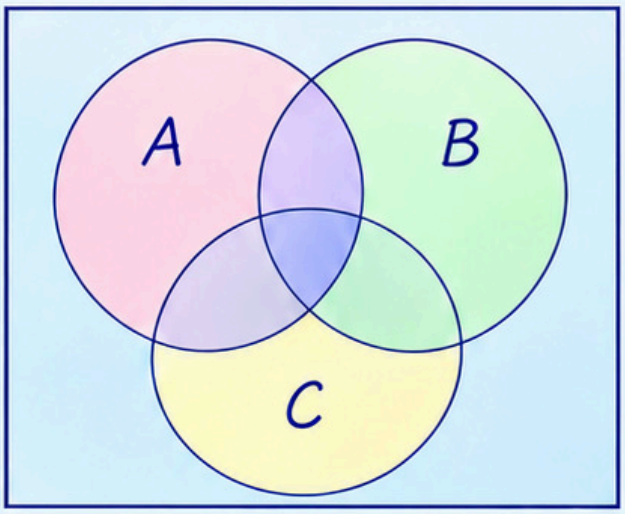


- At least one crop = 48 + 39 − 21 = 66
- Neither = 75 − 66 = 9
- Only chillies = 48 − 21 = 27
- Only turmeric = 39 − 21 = 18

2.3 Three groups: the formula, and why the centre is added back (1/2)

Three-set diagram

inside the rectangle, circle A at the top left, B at the top right and C below, each cutting the other two. The part common to all three is the centre. Around it lie three parts, each common to two circles only. Each circle also has an outer part of its own. The space outside all three circles is the eighth region.

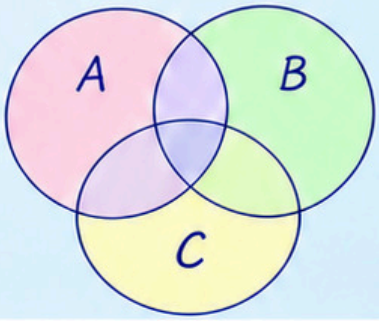


$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Why each step is needed:

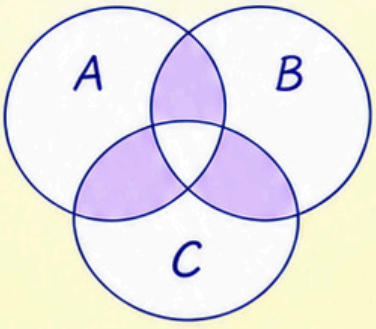
1. Add the three group totals.

A member of exactly one group is counted once; a member of exactly two groups, twice; a member of all three, three times.



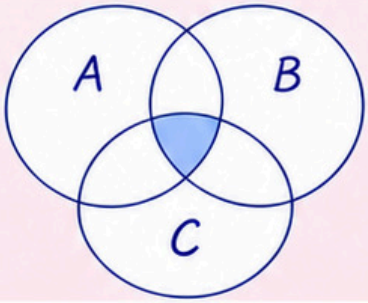
2. Subtract the three pair totals.

A member of exactly two groups is in one pair total, so the count falls from 2 to 1. A member of all three is in all three pair totals, so the count falls from 3 to 0: the centre has now dropped out.



3. Add the centre back once.

Now every member inside the circles is counted exactly once.



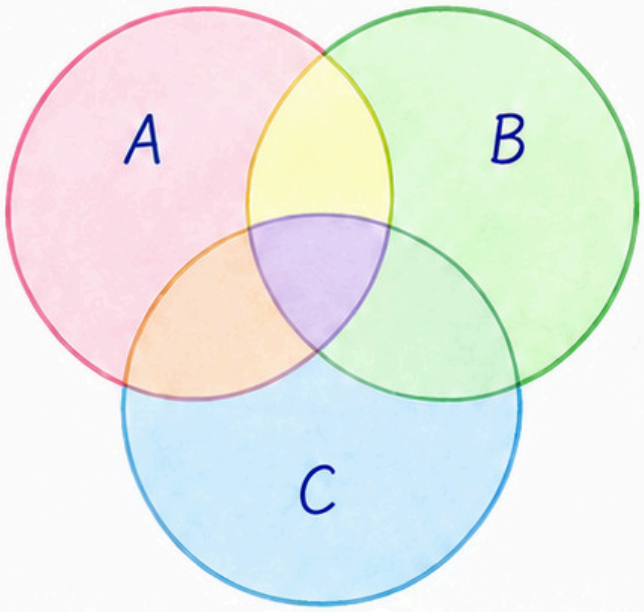
2.3 Three groups: the formula, and why the centre is added back (2/2)

A pair total, such as "20 like both tea and coffee", means everyone in both groups, including those in all three. It is the "two but not the third" region plus the centre.

Formula:

$$\text{None} = n(U) - n(A \cup B \cup C).$$

Venn diagram for three groups

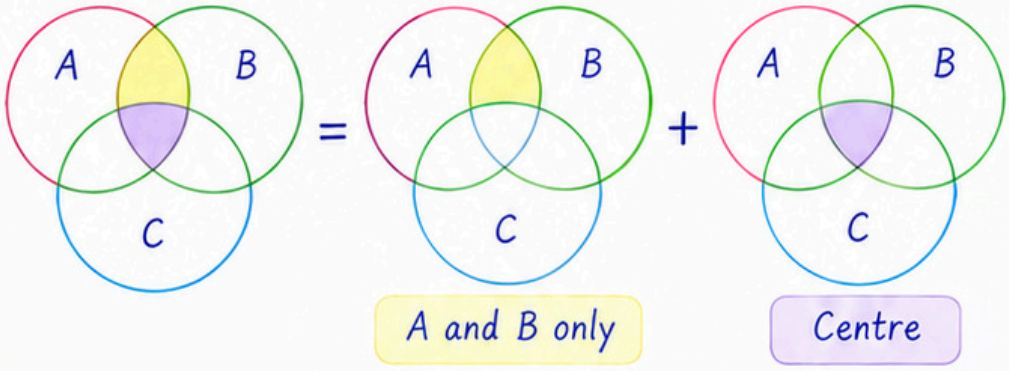


Regions

- Centre: in A, B and C.
- Two but not the third (e.g., in A and B only).
- Only in A.
- Only in B.
- Only in C.

Why the centre is added back

A pair total already includes the centre. So, "both A and B" = (A and B only) + (the centre).



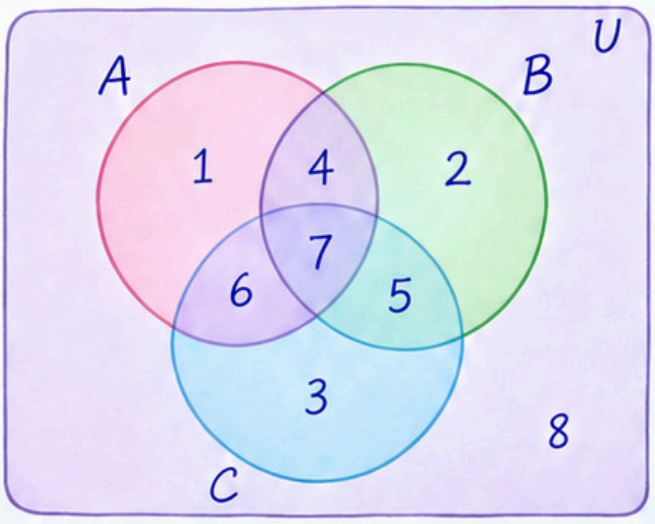
2.4 The eight regions, and formulas for "exactly one", "exactly two" and "at least two" (1/2)

Use these region numbers consistently.

The eight regions

- There are 8 regions with three sets A, B, C.
- Regions 1 to 7 are inside $A \cup B \cup C$.
- Region 8 is outside all three circles.

The region diagram



Formulas for useful counts

- Exactly one:
  $n(1) + n(2) + n(3)$
- Exactly two:
  $n(4) + n(5) + n(6)$
- At least two:
  $n(4) + n(5) + n(6) + n(7)$

The eight regions and how to find them

Region	Name in words	Found from the totals as
1	Only A	$n(A) - n(A \cap B) - n(C \cap A) + n(A \cap B \cap C)$
2	Only B	$n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$
3	Only C	$n(C) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
4	A and B but not C	$n(A \cap B) - n(A \cap B \cap C)$
5	B and C but not A	$n(B \cap C) - n(A \cap B \cap C)$
6	C and A but not B	$n(C \cap A) - n(A \cap B \cap C)$
7	All three (the centre)	$n(A \cap B \cap C)$
8	None (outside all three circles)	$n(U) - n(A \cup B \cup C)$

Why "only A" adds the centre back

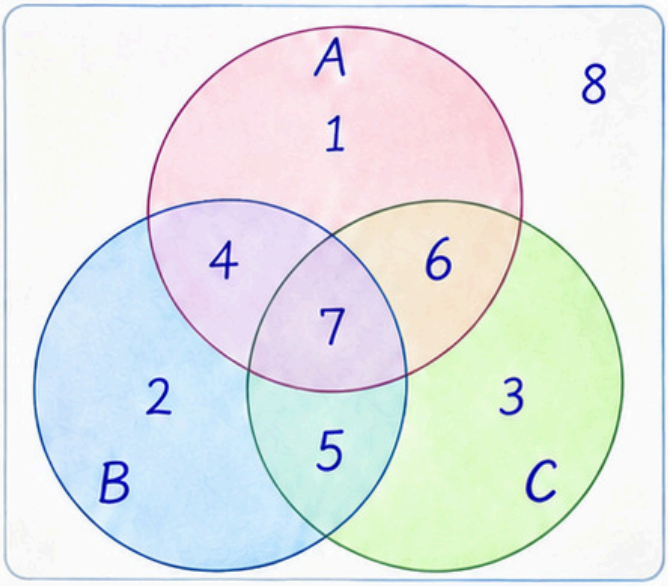
- $n(A) - n(A \cap B) - n(C \cap A)$ takes the centre out twice, but it is inside A only once.

2.4 The eight regions, and formulas for "exactly one", "exactly two" and "at least two" (2/2)

Write S_1 for the sum of the three group totals, S_2 for the sum of the three pair totals, and S_3 for $n(A \cap B \cap C)$.

The eight regions

- Three sets A, B and C divide the universal space into 8 regions.
- Label the regions as shown.



Useful totals in terms of regions

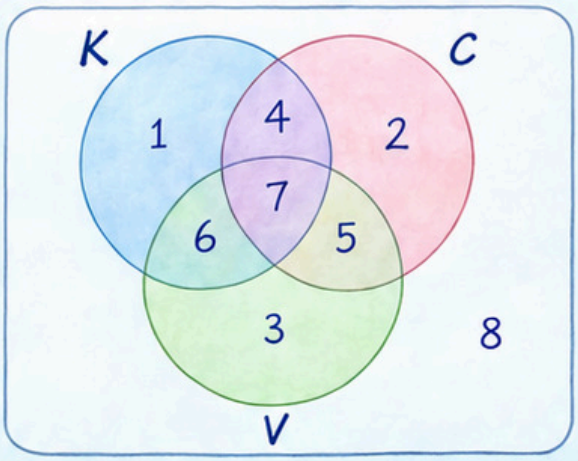
- r_i = number of members in region i (labels are not counts).
- $S_1 = r_1 + r_2 + r_3 + 2(r_4 + r_5 + r_6) + 3r_7$
- $S_2 = r_4 + r_5 + r_6 + 3r_7$
- $S_3 = r_7$

Phrase	Regions	Formula	Why
Exactly two	4, 5 and 6	$S_2 - 3 \times S_3$	Each pair total holds the centre once, so S_2 holds it three times
At least two	4, 5, 6 and 7	$S_2 - 2 \times S_3$	Exactly two, plus the centre once
Exactly one	1, 2 and 3	$S_1 - 2 \times S_2 + 3 \times S_3$	S_1 counts regions 4 to 6 twice and region 7 three times; take away $2 \times$ (exactly two) and $3 \times S_3$
At least one	1 to 7	$S_1 - S_2 + S_3$	The three-set formula

2.5 Filling the diagram from the centre outwards (1/2)

The centre lies inside every pair total and every group total. Once it is known, each "two but not the third" region is one subtraction, and each "only" region is one more.

Three-set Venn: region labels

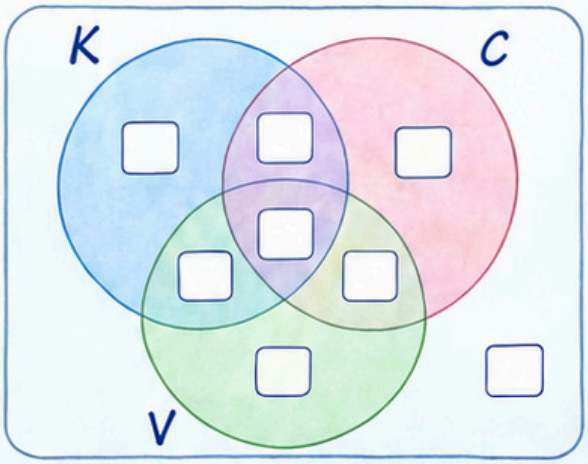


Steps to fill the diagram

1. Write the centre (region 7). ***If it is not given,*** find it from the number in none: at least one = $n(U)$ − none, and then S_3 = (at least one) − S_1 + S_2, from the three-set formula.
2. Each pair total minus the centre gives regions 4, 5 and 6.
3. Each group total minus its two "two but not the third" regions and the centre gives regions 1, 2 and 3.
4. None (region 8) = $n(U)$ − (regions 1 to 7).
5. **Check the data.** Every region must be 0 or more, and the eight regions must add up to $n(U)$. A negative region means the data are impossible, or a pair total has been read as "exactly two".

Example (centre not given):

of 150 students, 70 play kabaddi (K), 65 cricket (C) and 50 volleyball (V); 30 play K and C, 20 play C and V, 25 play K and V; 22 play none of the three.



2.5 Filling the diagram from the centre outwards (2/2)

At least one = 150 − 22 = 128.

S1 = 70 + 65 + 50 = 185 and

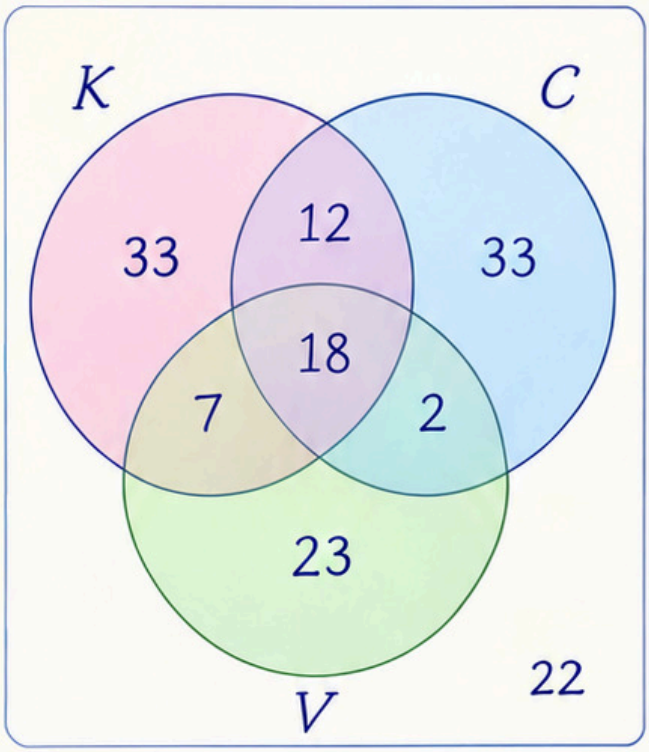
S2 = 30 + 20 + 25 = 75.

Centre = 128 − 185 + 75 = 18.

Method (centre outwards)

- Find the number in all three sets first.
- Then find the three pairwise-only regions.
- Next find the three only regions by subtraction.
- Finally, obtain the number in none.

Venn diagram (filled)



Region	Working	Students
All three	128 − 185 + 75	18
K and C but not V	30 − 18	12
C and V but not K	20 − 18	2
K and V but not C	25 − 18	7
Only K	70 − 12 − 7 − 18	33
Only C	65 − 12 − 2 − 18	33
Only V	50 − 2 − 7 − 18	23
None	Given	22

Check

- Check: the eight regions add up to 150, and none is negative.
- Exactly one = 33 + 33 + 23 = 89 = 185 − 2 × 75 + 3 × 18.

2.6 Two groups: the least and the greatest possible overlap (1/2)

When only $n(U)$, $n(A)$ and $n(B)$ are given, the overlap is not fixed. The question then asks for the least or the greatest number that can be in both.

Key symbols:

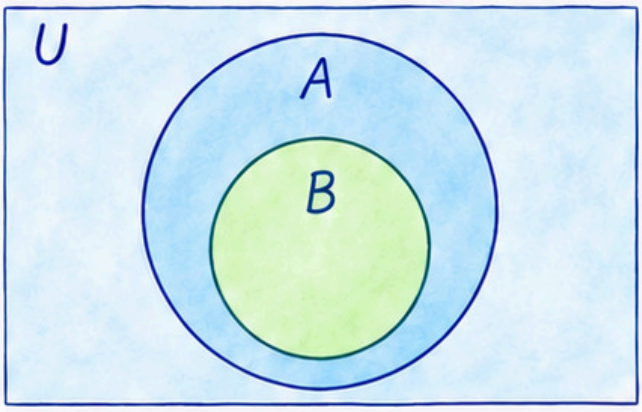
- U : the universal set (the whole group)
- A : the first group
- B : the second group

The results

Limit	Value	Why	When it happens
Greatest overlap	The smaller of $n(A)$ and $n(B)$	The common part lies inside each circle, so it cannot be larger than the smaller circle	The smaller circle lies wholly inside the larger one (a subset)
Least overlap	$n(A) + n(B) - n(U)$, if this is positive; otherwise 0	$n(A \cup B)$ cannot exceed $n(U)$, so $n(A \cap B) = n(A) + n(B) - n(A \cup B)$ is at least $n(A) + n(B) - n(U)$	No one is in neither; if $n(A) + n(B)$ is $n(U)$ or less, the circles can be drawn apart and the least is 0

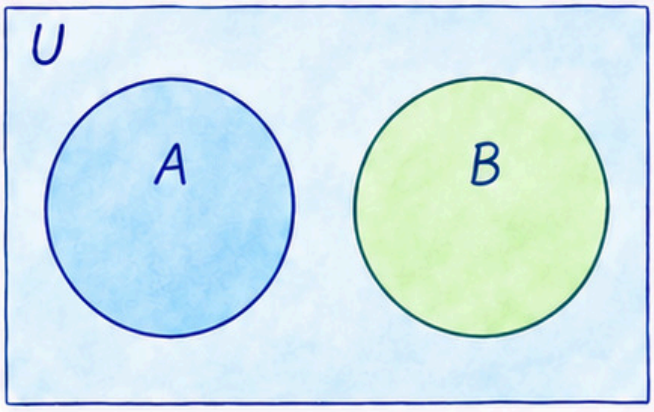
Diagrams (how the two cases can look)

Greatest overlap



The smaller circle lies wholly inside the larger one.

Least overlap



If $n(A) + n(B)$ is $n(U)$ or less, the circles can be drawn apart and the least is 0.

2.6 Two groups: the least and the greatest possible overlap (2/2)

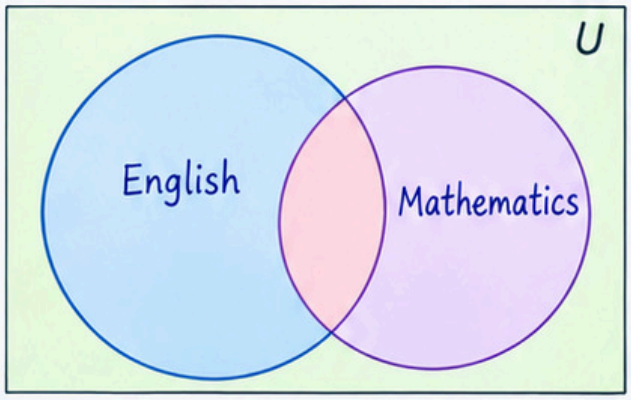
The room method for the least: place the larger group first. The room outside it is n(U) minus that group. Put as many of the smaller group there as will fit. The rest of the smaller group must lie inside the larger one, and that rest is the least overlap.

Example: of 100 students, 70 passed in English and 60 in Mathematics.

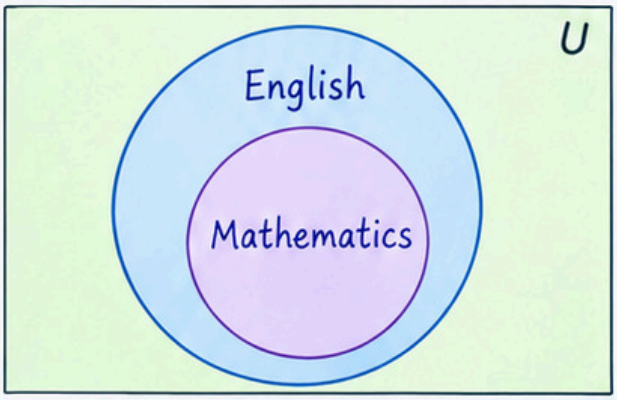
Least number who passed both: room outside English = 100 − 70 = 30, so at least 60 − 30 = 30 passed both.

Greatest: 60, when every student who passed in Mathematics also passed in English.

Least overlap (room method)



Greatest overlap

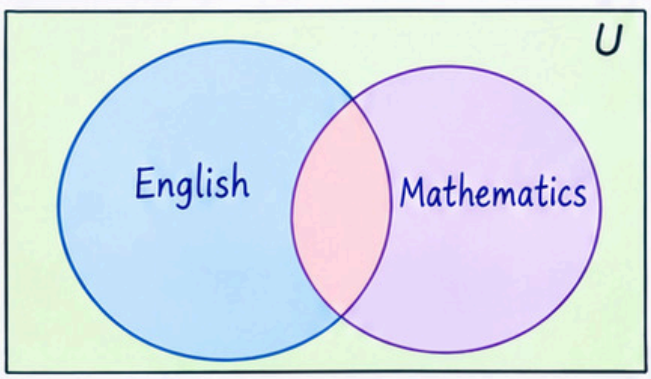


When some are known to be in neither.

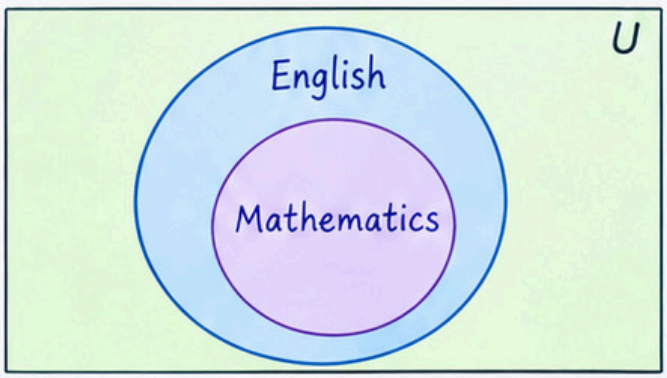
If at least k members are in neither, at most n(U) − k can be inside the circles, so use n(U) − k in place of n(U).

Example: if at least 10 of the 100 students passed in neither subject, at least 70 + 60 − 90 = 40 passed both. The greatest stays 60: with all 60 inside the English circle, 30 passed in neither, which is more than 10.

Least overlap with some in neither



Greatest overlap with some in neither



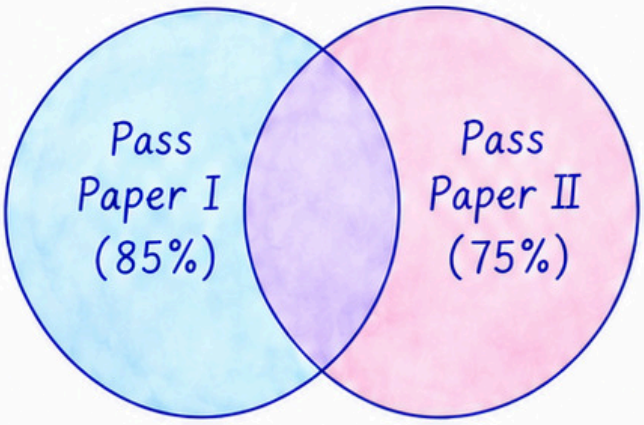
2.7 Percentage versions

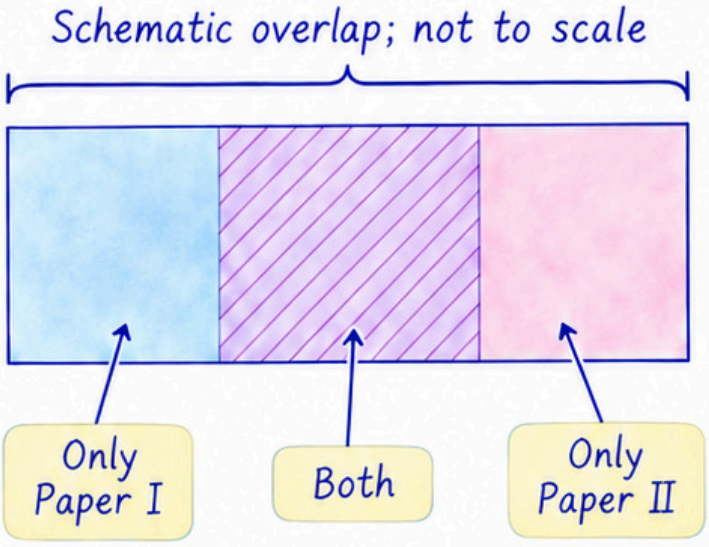
Take the total as 100 and use every formula above on the percentages; turn a percentage into a number only at the end (x% of the total).

Formulas (for two sets):

- At least: $P(A \cap B) \geq P(A) + P(B) - 100\%$
- At most: $P(A \cap B) \leq \min\{P(A), P(B)\}$
- Union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: if 85% of candidates passed Paper I and 75% passed Paper II, at least 85 + 75 − 100 = 60% passed both, and at most 75% did. Of 2,000 candidates, that is at least 1,200 and at most 1,500.





Takeaway: At fixed total: maximum overlap is the smaller group; minimum overlap is max{0, total of both groups − overall total}.

Extra section: a drill of eight questions with working (1/5)

Cover the working, solve, then check. The figures are illustrative.

Region check

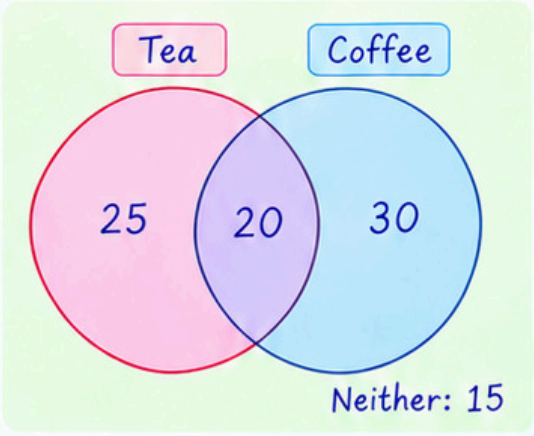
Each person must appear in exactly one region. Add all regions, including neither, to recover the total.

1. Of 90 people, 25 drink only tea, 30 drink only coffee and 15 drink neither. How many drink both, and how many drink tea?

Working:

The four regions add up to 90, so both = 90 − 25 − 30 − 15 = 20.
Tea = only tea + both
= 25 + 20 = 45.

Answer: 20 drink both; 45 drink tea.

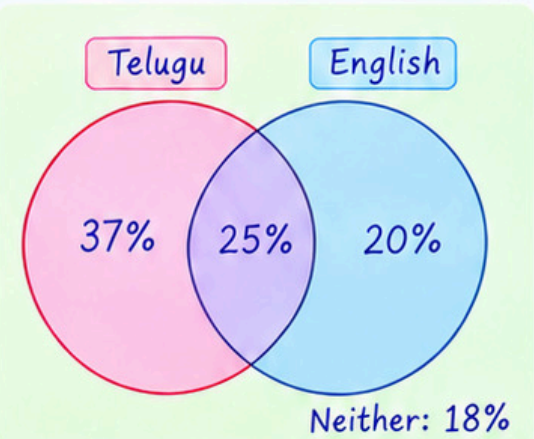


2. In a town, 62% of households read a Telugu newspaper, 45% read an English newspaper and 18% read neither. What percentage read both? If the town has 4,000 households, how many is that?

Working:

At least one = 100 − 18 = 82%.
Both = 62 + 45 − 82 = 25%.
25% of 4,000 = 1,000.

Answer: 25%, that is, 1,000 households.

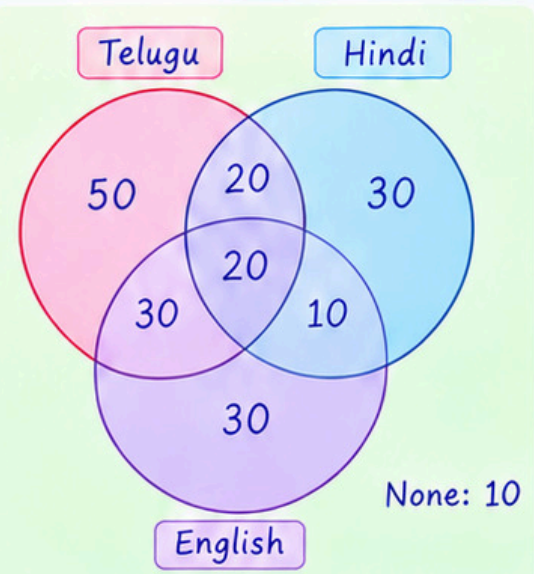


3. Of 200 employees, 120 speak Telugu, 80 Hindi and 90 English; 40 speak Telugu and Hindi, 30 Hindi and English, 50 Telugu and English; 20 speak all three. How many speak at least two of the languages, and how many speak none?

Working:

Exactly two:
Telugu & Hindi only = 40 − 20 = 20
Hindi & English only = 30 − 20 = 10
Telugu & English only = 50 − 20 = 30
At least two = 20 + 10 + 30 + 20 = 80.
At least one
= 120 + 80 + 90 − 40 − 30 − 50 + 20
= 190.
None = 200 − 190 = 10.

Answer: 80 speak at least two; 10 speak none.



Venn diagram drill: Questions 3 and 4

Question 3: working continued

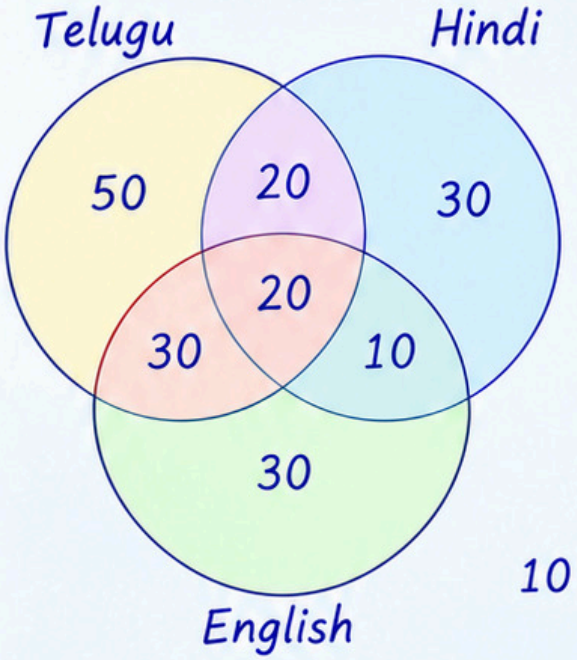
At least two $= S_2 - 2 \times S_3 =$
$(40+30+50) - 2 \times 20 = 80.$

Exactly two $= 20+10+30 = 60;$
plus centre20 gives80.

At least one $= 120+80+90-120+20 = 190.$

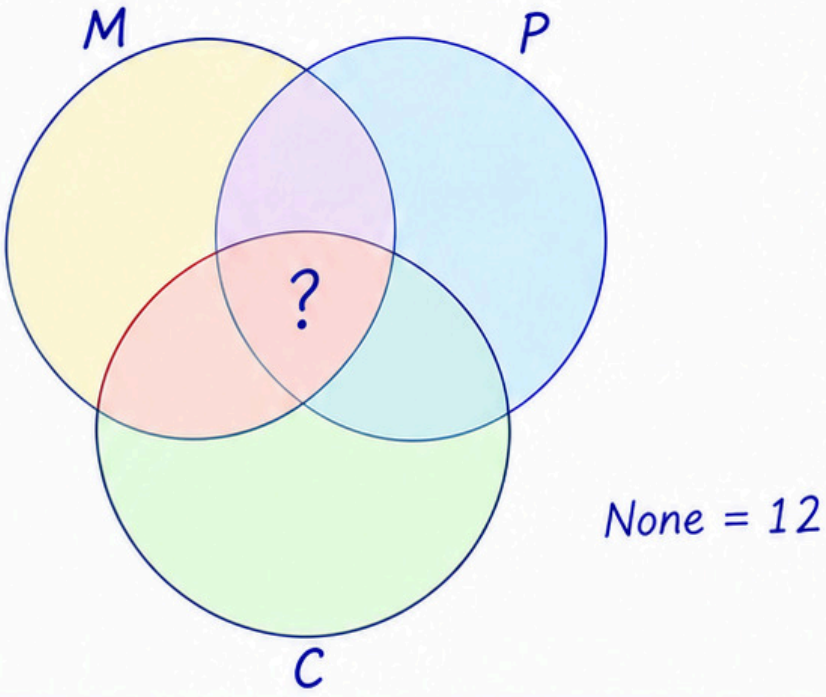
None $= 200 - 190 = 10.$

Answer: 80 speak at least two;
10 speak none.



Question 4: try before turning the page

Of 100 students, 45 study Mathematics (M), 40 Physics (P) and 35 Chemistry (C); 15 study M and P, 12 study P and C, 10 study M and C; 12 study none of the three. How many study all three, and how many study only Chemistry?



Venn diagram drill: Questions 4 and 5

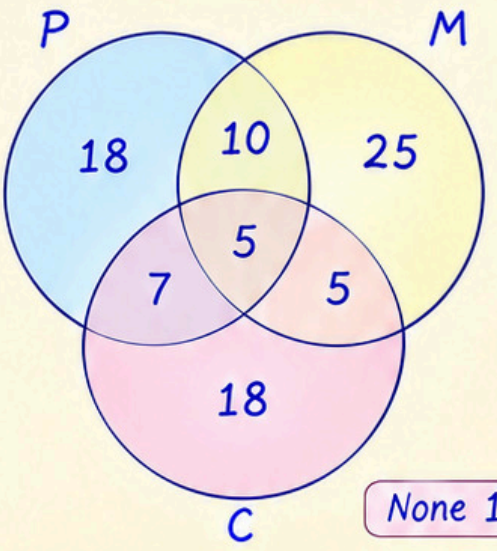
Question 4: working continued

Working:

at least one = 100 − 12 = 88. S1 = 120 and S2 = 37. Centre = 88 − 120 + 37 = 5. P and C but not M = 12 − 5 = 7; M and C but not P = 10 − 5 = 5. Only C = 35 − 7 − 5 = 18.

Check: the other regions are M and P but not C = 10, only M = 25 and only P = 18; the seven inside regions add up to 88, and none is negative.

Answer: 5 study all three; 18 study only Chemistry.



5. Of 80 students, 50 study Hindi and 45 study Sanskrit. What are the least and the greatest numbers who can study both? What is the greatest number who can study neither?

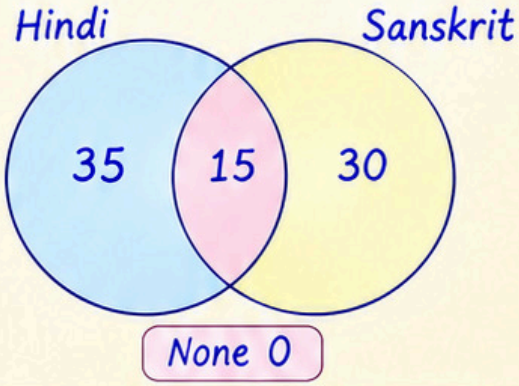
Working:

least = 50 + 45 − 80 = 15 (room outside Hindi = 30, and 45 − 30 = 15). Greatest = the smaller group = 45. Greatest neither = 80 − 50 = 30, when all 45 Sanskrit students also study Hindi.

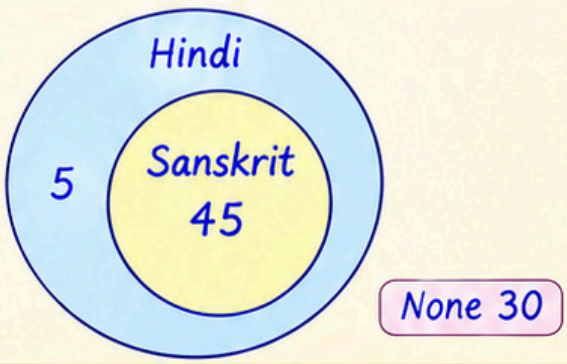
Answer:

least 15, greatest 45; at most 30 study neither.

Least-overlap diagram:



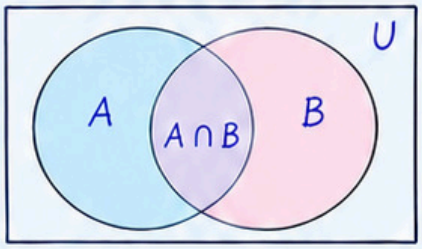
Greatest-overlap diagram:



Extra section: a drill of eight questions with working (4/5)

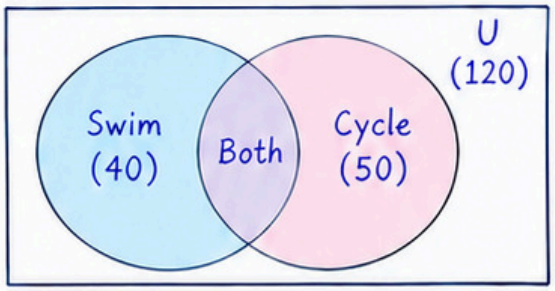
Use Venn diagrams and counting formulas to find least, greatest or exact numbers. For two groups: at least one = A + B − both.

Two groups (A, B):



Only A = A − A ∩ B
Only B = B − A ∩ B
At least one = A + B − A ∩ B
Neither = U − (A + B − A ∩ B)

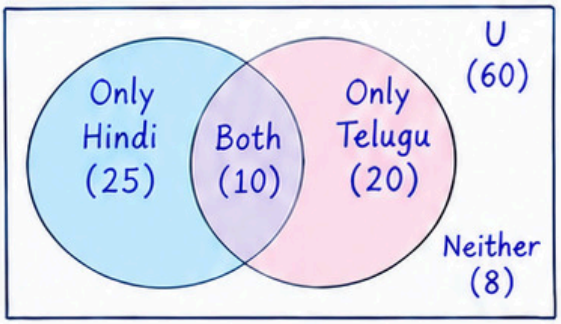
6. Of 120 people, 40 swim and 50 cycle. What are the least and the greatest numbers who can do both?



Working: 40 + 50 − 120 is negative, so the two groups can be wholly separate: least = 0. Greatest = 40. In this case at least 120 − 90 = 30 people do neither.

Answer: least 0, greatest 40.

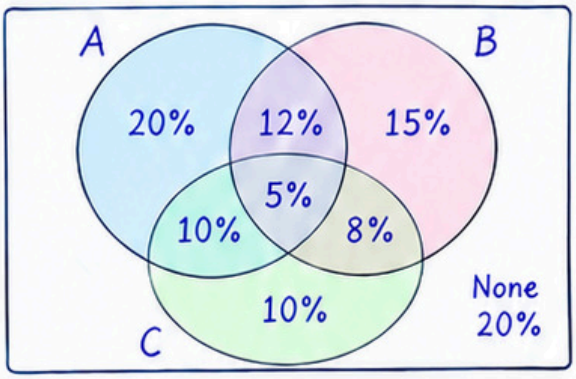
7. A report says: of 60 students, 35 like Hindi films, 30 like Telugu films, 10 like both and 8 like neither. Can the report be right?



Working: at least one by the formula = 35 + 30 − 10 = 55. At least one from "neither" = 60 − 8 = 52. The two do not agree, so the four regions would add up to 25 + 10 + 20 + 8 = 63, not 60.

Answer: no; the data are impossible.

8. In a survey of 600 people about three apps A, B and C, the regions in percentages are: only A 20%, only B 15%, only C 10%, A and B but not C 12%, B and C but not A 8%, C and A but not B 10%, all three 5%, none 20%. How many people use exactly two apps, and how many use at least two?



Working:
Exactly two apps = 12% + 8% + 10% = 30%.
= 30% of 600 = 180.
At least two = (12% + 8% + 10% + 5%) = 35%.
= 35% of 600 = 210.

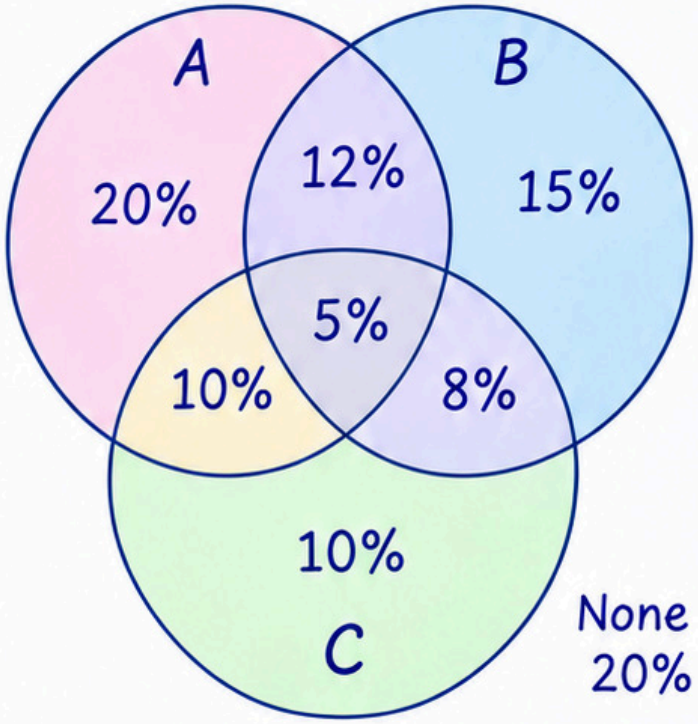
Answer: exactly two apps = 180;
at least two apps = 210.

Exactly two apps or at least two apps?

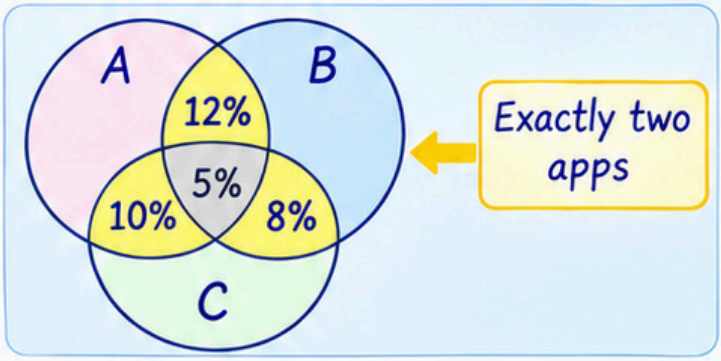
Question:

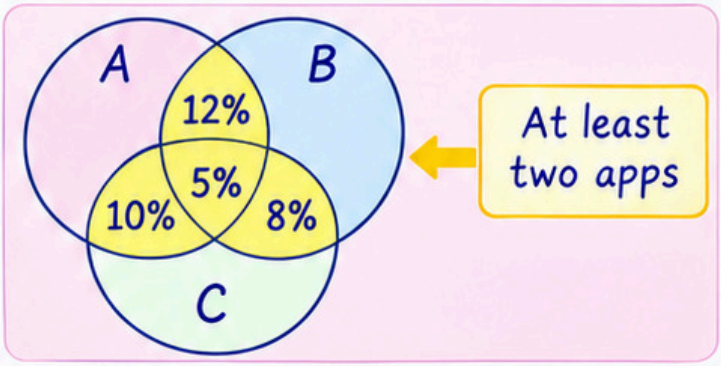
Among 600 people: only A 20%, only B 15%, only C 10%, A and B but not C 12%, B and C but not A 8%, C and A but not B 10%, all three 5%, none 20%.
How many use exactly two apps? How many use at least two?

Venn diagram:



Selection diagram:





Working:

Exactly two = 12% + 8% + 10% = 30%.
30% of 600 = 180.

At least two = 30% + 5% = 35%.
35% of 600 = 210.

Check: 20 + 15 + 10 + 12 + 8 + 10 + 5 + 20 = 100%.

Answer: 180 use exactly two apps; 210 use at least two apps.

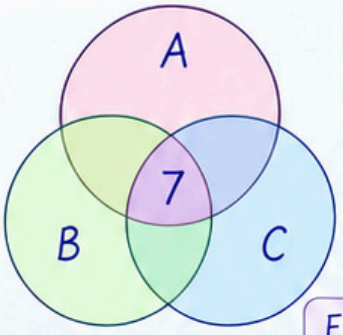
Three common exam traps

A pair total already contains the centre.

If 33 are in A and B, 25 in B and C, 19 in C and A, and 7 in all three, the pair totals add up to 77.

Exactly two = 77 − 3 × 7 = 56, not 77; at least two = 77 − 2 × 7 = 63.

Ask which one the question wants before you subtract.



A and B = 33
B and C = 25
C and A = 19

Total of pair totals = 77

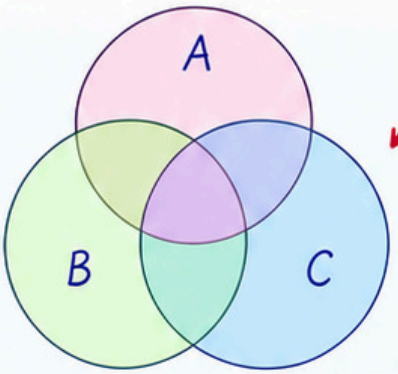
Exactly two = 77 − 3 × 7 = 56

At least two = 77 − 2 × 7 = 63

The seven regions inside the circles are not the whole group.

They add up to the number in at least one group. Those in none are outside every circle, so total = at least one + none.

When the centre has to be found, start from total minus none, and when the question asks for a region inside the circles, never let the none group creep into it.



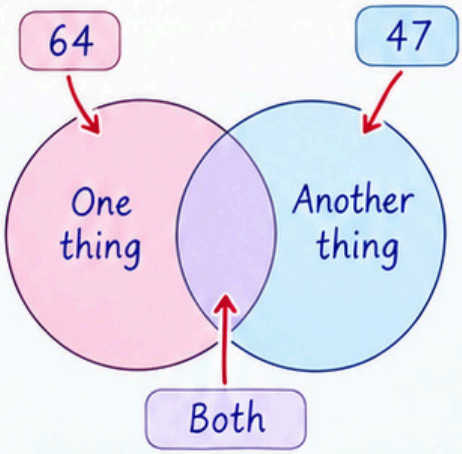
None (outside every circle)

Total = at least one + none

Least and greatest overlap are not interchangeable.

The greatest overlap is the smaller group; the least is the two group totals minus the total, or 0 only when that comes out negative.

Of 90 people, if 64 like one thing and 47 another, at least 21 and at most 47 like both; 0 is not possible.



At least 21 like both.

At most 47 like both.